Assignatura	Astrodynamics
CODI	220332

Activitat avaluable	%Ponderació nota final	Alumne	Nota	Pàgina
Exercici Entregable	10	Enunciat	-	2
		Resolució	-	3
		PB	5,0	4
		AT	8,0	7
		DQ	10	18
Examen Final	40	Enunciat i Correcció	-	34
		LN	3,1	42
		TR	6,6	49
		UG	7,2	57
		PM	9,8	65

Assignment 1 on Reference Frames

Compute the right ascension and declination of Cassini from Earth, during the Grand Finale re-entry into Saturn.

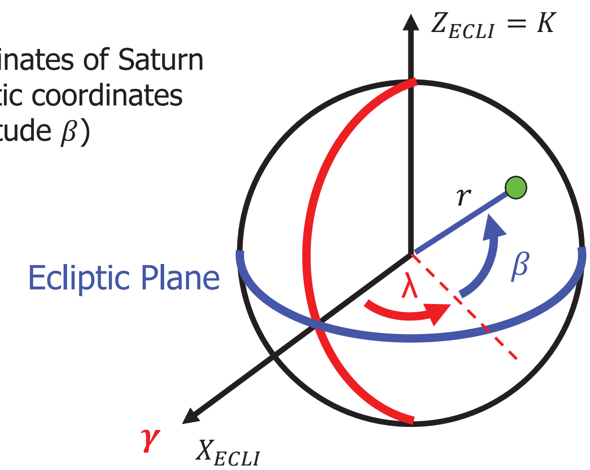
Deadline: *Partial Exam*
Maximum Pages: *4*
Groups of three

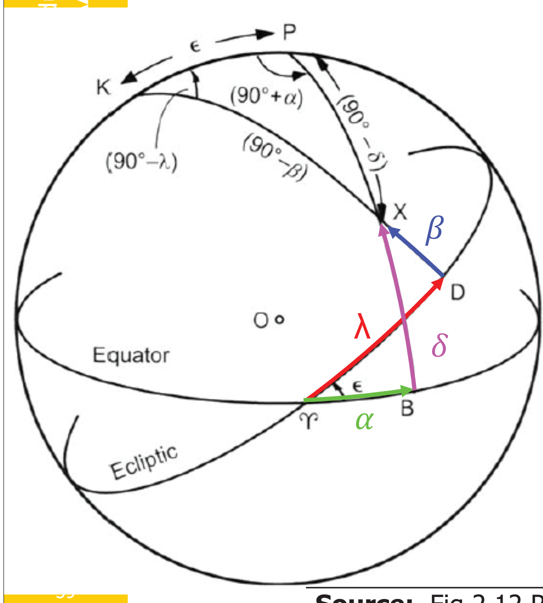
Source: https://solarsystem.nasa.gov/missions/cassini/mission/grand-finale/overview/

Assignment 1 on Reference Frames

1) Once the heliocentric ecliptic XYZ coordinates of Saturn have been computed, determine the ecliptic coordinates (i.e. celestial longitude λ and celestial latitude β)

$$x = r\cos\beta\cos\lambda$$
$$y = r\cos\beta\sin\lambda$$
$$z = r\sin\beta$$





2) Obtain the declination δ and the right ascension α solving the spherical triangle composed by
- Ecliptic Pole (K)
- North Pole (P)
- Saturn position (X)

$$\sin\delta = \sin\beta\cos\varepsilon + \cos\beta\sin\lambda\sin\varepsilon$$
$$\cos\delta\cos\alpha = \cos\beta\cos\lambda$$
$$\cos\delta\sin\alpha = -\sin\beta\sin\varepsilon + \cos\beta\sin\lambda\cos\varepsilon$$

Source: Fig 2.12 Roy AE (2005) "Orbital Motion" 4th Ed, Institute of Physics Publishing

1) Earth and Saturn state vectors (m) at 2017 9 15 10

a) Earth
rE=[149154157823.59161 -19595253613.626305 789195.69570630370]
vE=[3395.3531760092610 29423.115875348496 -1.1850112716553731]

b) Saturn
rS=[-77601953122.596344 -1500662823543.8748 29186774403.608196]
vS=[9119.9363531768449 -528.40138485647105 -353.66374117125878]

2) Relative Vector
r=rS-rE;
ru=r/norm(r);

3) Ecliptic coordinates (i.e. celestial longitude and celestial latitude)

$$x = r\cos\beta\cos\lambda$$
$$y = r\cos\beta\sin\lambda$$
$$z = r\sin\beta$$

$\beta = \text{asin}\left(\frac{z}{r}\right) = 1.1159^{\circ}$

$\lambda = atan\left(\frac{y}{x}\right) = -98.705^{\circ} = 261.295^{\circ}$

4) Assuming an ecliptic angle $= 23.439$, obtain δ and α by solving the spherical triangle composed by
- Ecliptic Pole
- North Pole
- Saturn position

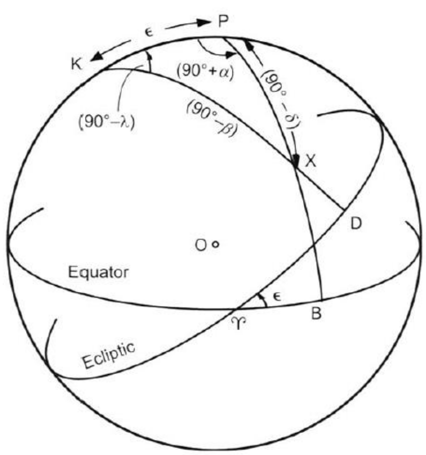


Fig 2.12 Roy (2005) “Orbital Motion” 4th Ed

$$\sin\delta = \sin\beta\cos\varepsilon + \cos\beta\sin\lambda\sin\varepsilon$$
$$\cos\delta\cos\alpha = \cos\beta\cos\lambda$$
$$\cos\delta\sin\alpha = -\sin\beta\sin\varepsilon + \cos\beta\sin\lambda\cos\varepsilon$$

Hence

$$\delta = asin(\sin\beta\cos\varepsilon + \cos\beta\sin\lambda\sin\varepsilon) = -22.04^{\circ}$$

$$\alpha = atan\left(\frac{-\sin\beta\sin\varepsilon + \cos\beta\sin\lambda\cos\varepsilon}{\cos\beta\cos\lambda}\right) = -99.39^{\circ}$$
$$= 260.61^{\circ} = 17\text{h } 22\text{m } 26\text{ s}$$

PB

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH

Assignment 1 on reference frames

The aim of this report is to compute the right ascension and the declination of the Cassini probe from the Earth during the Grand Final re-entry into Saturn's atmosphere. The Grand Final is the final phase of the Cassini mission that has brought Cassini probe close to Saturn to observe from a never seen angle the planet and its rings before dying in its atmosphere.

To simplify the calculation, we will consider that the position of Cassini probe and Saturn are the same because the difference of position between the two from the Earth is negligeable. Then we only have to compute the right ascension and declination of Saturn.

The first step is to compute the orbital elements of Saturn's orbit at the date of 15th September 2017 which is the date of re-entry of Cassini in the planet's atmosphere.

We first calculate the Julian Date Number and Julian Centuries of the 15th September 2017:

Julian day (15 September 2017 19h): 2458012.2916666665
Julian centuries: 0.17706479580195789

We can then propagate the orbital elements of Saturn to the Julian Centuries given by the following table:

	a (AU) $\dot{a}$ (AU/Cy)	e $\dot{e}$ (1/Cy)	i (°) $\dot{i}$ (°/Cy)	Ω (°) $\dot{\Omega}$ (°/Cy)	ϖ (°) $\dot{\varpi}$ (°/Cy)	L (°) $\dot{L}$ (°/Cy)
Mercury	0.38709927	0.20563593	7.00497902	48.33076593	77.45779628	252.25032350
	0.00000037	0.00001906	−0.00594749	−0.12534081	0.16047689	149,472.67411175
Venus	0.72333566	0.00677672	3.39467605	76.67984255	131.60246718	181.97909950
	0.00000390	−0.00004107	−0.00078890	−0.27769418	0.00268329	58,517.81538729
Earth	1.00000261	0.01671123	−0.00001531	0.0	102.93768193	100.46457166
	0.00000562	−0.00004392	−0.01294668	0.0	0.32327364	35,999.37244981
Mars	1.52371034	0.09339410	1.84969142	49.55953891	−23.94362959	−4.55343205
	0.0001847	0.00007882	−0.00813131	−0.29257343	0.44441088	19,140.30268499
Jupiter	5.20288700	0.04838624	1.30439695	100.47390909	14.72847983	34.39644501
	−0.00011607	−0.00013253	−0.00183714	0.20469106	0.21252668	3034.74612775
Saturn	9.53667594	0.05386179	2.48599187	113.66242448	92.59887831	49.95424423
	−0.00125060	−0.00050991	0.00193609	−0.28867794	−0.41897216	1222.49362201
Uranus	19.18916464	0.04725744	0.77263783	74.01692503	170.95427630	313.23810451
	−0.00196176	−0.00004397	−0.00242939	0.04240589	0.40805281	428.48202785
Neptune	30.06992276	0.00859048	1.77004347	131.78422574	44.96476227	−55.12002969
	0.00026291	0.00005105	0.00035372	−0.00508664	−0.32241464	218.45945325
(Pluto)	39.48211675	0.24882730	17.14001206	110.30393684	224.06891629	238.92903833
	−0.00031596	0.00005170	0.00004818	−0.01183482	−0.04062942	145.20780515

Reproduced with permission from Standish et al. (2013).

This gives us the following orbital elements propagated to Julian Centuries:

- Semi-major axis: a = 9.53645450276637 UA = 1426633287555.448 m
- Eccentricity: e = 0.05377150288997262
- Inclination: i = 2.486334683380504 °
- Longitude of the ascending node: Ω = 113.61130977950137 °
- Argument of the periastre: ϖ = 92.5246930900429 °
- Mean longitude: L = 266.41482778039654 °

We then calculate the angular momentum with the formula:

Angular momentum: $h = \sqrt{\mu_s a(1-e^2)}$ = 1123364.0571139366

We can deduce the argument of perihelion and the mean anomaly

Argument of the perihelion: ω = $\varpi - \Omega$ = 338.9133833105415 °

Mean anomaly: M = L$-$ ϖ = 173.89013469035365 °

We then implement a solver of the Kepler equation to calculate the eccentric anomaly. Knowing that the Kepler equation is $M = E - esin(E)$, we solve this by using the Newton-Raphson iterative method:

$$E_{i+1} = E_i + \frac{E_i - esin(E_i) - M}{1 - ecos(E_i)}$$

This gives us the result in radians after few iterations:

Eccentric anomaly: E = [3.034955387083081, 3.04038810581619, 3.0403880270031536, 3.0403880270031536, 3.0403880270031536, 3.0403880270031536, 3.0403880270031536, 3.0403880270031536]

The result is in radians because Python software use radians to make trigonometric calculations.

We can then deduce the true anomaly via the formula:

$$\theta = 2\,arctan\left(\frac{tan\left(\frac{E}{2}\right)}{\sqrt{\frac{1-e}{1+e}}}\right)$$ = 3.0456828572531816 rad = 171,88733853967335108 °

From those orbitals elements we can deduce the state vector with the following formulas:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = R \begin{pmatrix} r\cos(\theta) \\ rsin(\theta) \\ 0 \end{pmatrix}$$

$$with\ R = \begin{pmatrix} \cos(\Omega)\cos(w) - \sin(\Omega)\cos(i)\sin(w) & -\cos(\Omega)\sin(w) - \sin(\Omega)\cos(i)\cos(w) & \sin(\Omega)\sin(i) \\ \sin(\Omega)\cos(w) + \cos(\Omega)\cos(i)\sin(w) & -\sin(\Omega)\sin(w) + \cos(\Omega)\cos(i)\cos(w) & -\cos(\Omega)\sin(i) \\ \sin(i)\sin(w) & \sin(i)\cos(w) & \cos(i) \end{pmatrix}$$

$$and\ r = \frac{h^2}{\mu}\frac{1}{1+ecos(\theta)}\begin{pmatrix}\cos(\theta)\\ \sin(\theta)\\ 0\end{pmatrix}$$

The computation finally gives us:

$$r = \begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}-7.73064557 * 10^{10}\ m\\ -1.50067990 * 10^{12}\ m\\ 2.91753118 * 10^{10}\ m\end{pmatrix}$$

Once we have the state vector of Saturn from the Earth, we can deduce the celestial longitude λ and celestial latitude β.

$$\beta = sin^{-1}\left(\frac{z}{r}\right) = 1.1122950980934008\,°$$

$$\lambda = cos^{-1}\left(\frac{x}{rcos(\beta)}\right) = 92.94894452763805\,°$$

We can finally compute the declination δ and the right ascension α by solving the spherical triangle with the following formulas:

$$\delta = sin^{-1}(\sin(\beta)\cos(\varepsilon) + \cos(\beta)\sin(\lambda)\sin(\varepsilon))$$

$$\alpha = cos^{-1}\left(\frac{\cos(\beta)\cos(\lambda)}{\cos(\delta)}\right)$$

where ε is the angle between the equator and the ecliptic plane of the Earth. After doing some research we found that $\varepsilon = 23.26°$.

This gives us a final result of:

$$\boldsymbol{Declination}: \boldsymbol{\delta = 24.33941179392905}\,°$$

$$\textbf{Right ascension}: \boldsymbol{\alpha = 93.2362970698289}\,°$$

POLYTECHNIC UNIVERSITY OF CATALONIA – BARCELONA TECH (UPC)

ESCOLA SUPERIOR D'ENGINYERIES INDUSTRIAL, AEROESPACIAL I AUDIOVISUAL DE TERRASSA, ESEIAAT

MSc. in SPACE AND AERONAUTICAL ENGINEERING (MASE)

Course: ASTRODYNAMICS (Q1, 220332)

Computation of the Right Ascension and Declination of Cassini From Earth, during the Grand Finale Re-Entry into Saturn

---

Assignment 1: Right Ascension and Declination

AT

Terrassa, Barcelona, November 2020

Table of Contents

1. Introduction to Cassini and its Grand Finale Mission

Cassini was a NASA spacecraft tasked to explore Saturn and its icy moons. Cassini spent 20 years in space – 13 of which was exploring Saturn until it exhausted all of its fuel supply. To protect the icy moons, which has the potential to harbour life, Cassini was tasked with its final mission. This mission was to plunge itself into Saturn's atmosphere where it will return science data throughout, the mission was therefore named The Grand Finale. The Grand Finale mission began on the 22nd April 2017 and ended on the 15th September 2017 [1].

2. Introduction to Right Ascension and Declination

The right ascension and declination, similar to latitude and longitude, determines the position of an object with reference to a spherical body. The right ascension and declination describes the location of a celestial body in the Earth's sky. The right ascension is measured along the celestial equator, a projection of the equatorial plane. It's origin is determined by the vernal equinox and measured in the eastern direction. Declination is measured along the meridian where North is positive and South is negative measured from the celestial equator [2].

3. Determining the Right Ascension and Declination of Cassini

The right ascension and declination of Cassini can be assumed to be the same as that of Saturn's. This assumption can be backed up by taking into account the distance between Earth and Saturn and the altitude of Cassini at the beginning of the mission and computing the trigonometric angle. On the 23rd April Cassini came within 2950km of Saturn's 1 bar atmosphere [3]. The distance between Earth and Saturn is 1.2 billion km [4]. Using the trigonometric ratios an estimation of the angle can be determined by the following solution.

$$\tan(c) = \frac{2950}{1.2 * 10^{9}} = 2.4583 * 10^{-6}$$

$$c = 1.409 * 10^{-4} \text{ degrees}$$

This confirms the angle between Cassini and Saturn is negligible and that the right ascension and declination of Cassini can be assumed to be the same as Saturn's.

3.1. Orbital Elements of Earth and Saturn

Orbital elements define a bodies orbit around a given mass. To determine the position of Saturn and Earth both orbital elements are defined in the heliocentric reference frame and by the elements: semi-major axis, a, eccentricity, e, inclination, i, right ascension of the ascending nodes, Ω, the longitudinal of perihelion, ϖ, and the mean longitude, L. Table 1 displays the orbital elements at J2000 and how they change per Julian century, Cy.

	a, AU $\dot{a}$, AU/Cy	e $\dot{e}$, 1/Cy	i, deg $\dot{i}$, °/Cy
Earth	1.00000261 0.00000562	0.01671123 -0.00004392	-0.00001531 -0.01294668
Saturn	9.53667594 -0.00125060	0.05386179 -0.0005099	2.48599187 0.00193609

	Ω, deg $\dot{\Omega}$, °/Cy	ϖ, deg $\dot{\varpi}$, °/Cy	L, deg $\dot{L}$, °/Cy
Earth	0 0	102.93768 0.3232736	100.4645716 35999.37245
Saturn	113.66242448 -0.28867794	92.598878 -0.418972	49.95424423 1222.493622

Table 1: Heliocentric Orbital Elements [2]

From table 1 the orbital elements on the 15th September 2017 can be obtained by calculating the number of Julian days between 15th September 2017 Julian day J2000, T_0. The following algorithm displays how to compute the orbital elements.

Algorithm 1: Determining the heliocentric orbital elements of Earth and Saturn [5]

1) Compute the 15th September 2017 in Julian days, JD

   Where: J_0 measured the Julian day till noon. For example at J2000 equates to the 1st January 2000 12:00 which equates to a Julian day of 2451545.0, on the same day but at 00:00 the Julian day is 2451544.5 and on 2nd January 2000 at 00:00 the Julian day is 2451545.5. Thus, the time needs to be accounted for, *UT*.

$$JD = J_0 + \frac{UT}{24} \quad (1)$$

J_0 is computed using the following equation:

$$J_0 = 367y - INT\left[\frac{7*(y + INT\left(\frac{m+9}{12}\right)}{4}\right] + INT\left(\frac{275m}{9}\right) + d + 1{,}721{,}013.5 \quad (2)$$

Where: *y* is the year, *m* is the month, *d* is the day and *INT*() denoted acquiring the integer, rounding down. It must be stated that J_0 computes the Julian day to 00:00am. For the 15th September 2017 3:31am.

$$J_0 = 2458011.5 \text{ days}$$

$$UT = 3 + \frac{31}{60} = -8.4833 \text{ hours}$$

$$JD = 2458011.1465 \text{ days}$$

2) Compute the ratio of the difference of the 15th September 2017 and J2000 to the Julian century.

$$T_0 = \frac{JD - J2000}{Cy} \quad (3)$$

$$T_0 = \frac{2458011.1465 - 2451545}{36525}$$

$$= 0.177047$$

3) Compute the orbital elements of Earth and Saturn on 15th September. If ϕ represents any given orbital element then the orbital element is equated by using:

$$\phi = \phi_0 + \dot{\phi}T_0$$

	a, km	e	i, °
Earth	$1.4960*10^8$	0.01670	359.9977
Saturn	$1.4266*10^9$	0.0538	2.4863

	Ω, °	ϖ, °	L, °
Earth	0	102.9949	354.5574
Saturn	113.6113	92.5247	266.3765

Table 2: Orbital Elements at JD

Table 2 takes into consideration of the conversion between the astronautical unit to kilometres ($1.49597871*10^8$ km).

4) Obtain the angular momentum, h, from the semi-major axis and the eccentricity at JD

$$h = \sqrt{\mu a(1 - e^2)} \quad (4)$$

where μ is the gravitational parameter of the orbit's focus i.e. the Sun.

Earth:

$$h_E = 4.4551 * 10^9 \text{ kg.km}^2\text{.s}^{-1}$$

Saturn:

$$h_S = 1.3740 * 10^{10} \text{ kg.km}^2\text{.s}^{-1}$$

5) From the longitudinal of perihelion, ϖ, and the mean longitude, L, compute the argument of perihelion, ω, and the mean anomaly, M.

$$\omega = \varpi - \Omega \quad (5)$$

$$M = L - \varpi \quad (6)$$

If calculated values are either over 360º or less than 360º the values have to be adjusted to be in the range of -360º < ϕ < 360º.

Earth:

$$\omega_E = 102.9949 \text{ degrees}$$
$$M_E = 251.0553 \text{ degrees}$$

Saturn:

$$\omega_S = 338.9134 \text{ degrees}$$
$$M_S = 173.8518 \text{ degrees}$$

6) Implementing the mean anomaly, M and eccentricity, e, the eccentric anomaly, E, can be calculated using the Kepler's equation. Kepler's

$$M = E - esin(E) \quad (7)$$

Earth:

$$E_E = 250.5468 \text{ degrees}$$

Saturn:

$$E_S = 173..8575 \text{degrees}$$

7) Finally, calculate the true anomaly, θ, using the following equation.

$$\tan\left(\frac{E}{2}\right) = \sqrt{\frac{1-e}{1+e}}\tan\left(\frac{\theta}{2}\right) \quad (8)$$

Earth:

$$\theta_E = 249.6468 \text{ degrees}$$

Saturn:

$$\theta_S = 174.1788 \text{ degrees}$$

3.2. Determining the orbital state vectors of Earth and Saturn from the known orbital elements

Orbital vector can be obtained by implementing the true anomaly, θ, and the eccentricity, e, the angular momentum, h, and the gravitational constant, μ. The orbital vector is computed using the following equation:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{h^2}{\mu}\frac{1}{1+ecos(\theta)}\begin{pmatrix} cos\theta \\ sin\theta \\ 0 \end{pmatrix} \quad (9)$$

Earth:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_E = \begin{pmatrix} -5.2321 * 10^7 \\ -1.4104 * 10^8 \\ 0 \end{pmatrix}\text{km}$$

Saturn:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_S = \begin{pmatrix} -1.4952 * 10^9 \\ 1.5243 * 10^8 \\ 0 \end{pmatrix} \text{km}$$

Shown above are the coordinated of Earth and Saturn if plotted perifocal frame centred at the sun. Thus, the orientation of their orbits needs to be taken into account. To do this a rotational matrix, $\boldsymbol{R}$, is implemented taking into consideration the orbits, right ascension of ascending nodes, Ω, argument of perihelion, ω and the inclination, i. The negative of these angles are taken as a result of the natural clockwise direction of the Euler rotation matrix.

$$\boldsymbol{R}(-\Omega) = \begin{pmatrix} \cos(\Omega) & -\sin(\Omega) & 0 \\ \sin(\Omega) & \cos(\Omega) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\boldsymbol{R}(-i) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(i) & -\sin(i) \\ 0 & \sin(i) & \cos(i) \end{pmatrix}$$

$$\boldsymbol{R}(-\omega) = \begin{pmatrix} \cos(\omega) & -\sin(\omega) & 0 \\ \sin(\omega) & \sin(\omega) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\boldsymbol{R} = \boldsymbol{R}(-\Omega) * \boldsymbol{R}(-i) * \boldsymbol{R}(-\omega) \qquad (10)$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \boldsymbol{R} * \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Thus, the orbital state vectors are:
Earth:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_E = \begin{pmatrix} 1.4919 * 10^8 \\ -1.9266 * 10^7 \\ 775.8471 \end{pmatrix} \text{km}$$

Saturn:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_S = \begin{pmatrix} -8.5834 * 10^7 \\ -1.5002 * 10^9 \\ 2.9506 * 10^7 \end{pmatrix} \text{km}$$

3.3. Obtaining the right ascension and declination

As the right ascension and declination is a measurement relative to the position of Earth, the coordinates of Saturn needs to be related to the position of the Earth, R.

$$R = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_S - \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_E$$

From R the celestial coordinates can be obtained (i.e. the celestial longitude, λ and the celestial latitude, β). These values are determined by using the following equations.

$$X = rcos(\beta)\cos(\lambda) \qquad (11)$$

$$Y = rcos(\beta)\sin(\lambda) \qquad (12)$$

$$Z = rsin(\beta) \qquad (13)$$

$$r = \sqrt{X^2 + Y^2 + Z^2} \qquad (14)$$

Analytical solution: To account for the possibility of a negative angle the inverse sine function is used.

$$r = 1.5010 * 10^9 \text{km}$$

$$\beta = \sin^{-1}\left(\frac{Z}{r}\right) = 1.1273 \text{ degrees}$$

$$\lambda = \sin^{-1}\left(\frac{Y}{r * \cos(\beta)}\right) = -\ 80.9821 \text{ degrees}$$

The celestial coordinates are then used to compute the declination and the right ascension of the celestial body. The celestial coordinates are referenced to the Earth's axis of rotation and the ecliptic plane, as such, the obliquity of the ecliptic needs to be taken into account. The obliquity of the ecliptic is the angle between the ecliptic plane and celestial equator plane, which is currently 23.5º and denoted as ε. The following equations are used to determine the right ascension, α and declination δ.

$$\sin(\delta) = \sin(\beta)\cos(\varepsilon) + \cos(\beta)\cos(\lambda)\sin(\varepsilon) \quad (15)$$

$$\cos(\delta)\cos(\alpha) = \cos(\beta)\cos(\lambda) \qquad (16)$$

$$\cos(\delta)\sin(\alpha) = -\sin(\beta)\sin(\varepsilon) + \cos(\beta)\sin(\lambda)\cos(\varepsilon) \qquad (17)$$

Analytical solutions:

$$\delta = \sin^{-1}\begin{pmatrix} \sin(\beta)\cos(\varepsilon) \\ + \cos(\beta)\cos(\lambda)\sin(\varepsilon) \end{pmatrix}$$

$$= -22.0677 \text{ degrees}$$

$$\alpha = \sin^{-1}\left(\frac{-\sin(\beta)\sin(\varepsilon) + \cos(\beta)\sin(\lambda)\cos(\varepsilon)}{\cos(\delta)}\right)$$
$$= -80.2644 \text{ degrees}$$

3.4. Unit Conversion

To finalise the calculation, more suitable units are selected. The right ascension is converted to hours, minutes and seconds and declination is converted to degrees, minutes and seconds.

3.4.1. Right Ascension Unit Conversion Algorithm

As previously stated the Right ascensions units are represented in hours, minutes and seconds. The following algorithm describes how to convert degrees to hours, minutes and seconds.

1) Right ascension is measured from the vernal equinox in the Eastern direction. Thus if the angle computed is in the range of -360º ≤ α ≤ 0º then 360º is added to α. If 0º ≤ α ≤ 360º then move straight to step 2.
2) A full rotation around the celestial is 360º equivalent to 24 hours, therefore 1 hour is equal to 15º. Dividing α by 15º acquires the hour plus a decimal, the hour is represented by the integer. For example if the

computed hour decimal is 13.8974 then the hour is 13.

3) The remaining decimal (using the previous example), 0.8974 is multiplied by 60 (minutes). Following the same procedure, the obtained value is the minute decimals, thus the integer is deemed as the minute. Continuing with 0.8974 the minute calculated is 53 minutes 0.844 remaining.
4) Finally, to compute the seconds the remaining decimal from step 3 is multiplied by 60 (seconds). The integer is deemed as the seconds and the remaining decimal is unlisted. E.g. 0.844 is equivalent to 50 seconds.

Using the above algorithm the right ascension is:

$$\alpha = 18 \text{ hours, } 38 \text{ minutes and } 56 \text{ seconds}$$

3.4.2. Declination Unit Conversion Algorithm

As previously stated the declination is represented in the format of degrees, minutes and seconds. The following algorithm describes how to convert the degrees decimal to degrees, minutes and seconds:

1) From the degree decimal the integer is denoted as the degree and the remaining decimal is considered as the minute decimal. For example 13.8964º is considered as 13º and 0.8964 is the minute decimal.
2) The minute is determined by multiplying the minute decimal by 60. The integer of the computed value is the minute whilst the remaining decimal is the seconds decimal. E.g. 0.8964 is represented by 53 seconds and 0.844 is the seconds decimal.
3) Finally, the seconds is computed by multiplying the seconds decimal by 60. The seconds is represented by the integer. E.g. 0.844 is represented by 50 seconds.

Thus, declination is:

$$\delta = -22.0677^0 = -22^{\circ}\, 4'\, 3''$$

4. Conclusion

In conclusion on the 15th September 2017 the right ascension and declination of Saturn is computed as 18 hours, 38 minutes and 56 seconds -22º 4' 3". From comparisons to other computations there is an estimated error of 7.34% in the right ascension and an error of 1.54% in declination. This error is generated from the rotation matrix applied to the perifocal coordinates of Saturn, potentially a consequence of small errors being generated at the start of the calculation. These errors then propagate through the code and grow larger and larger.

5. Table of Equations

6. References

1. Overview | The Grand Finale – NASA Solar System Exploration [Internet]. NASA Solar System Exploration. 2020 [cited 7 November 2020]. Available from: https://solarsystem.nasa.gov/missions/cassini/mission/grand-finale/overview/

2. Curtis H. Orbital mechanics for engineering students. 4th ed. Elsevier; 2020.

3. Orbit Guide | The Grand Finale – NASA Solar System Exploration [Internet]. NASA Solar System Exploration. 2020 [cited 7 November 2020]. Available from: https://solarsystem.nasa.gov/missions/cassini/mission/grand-finale/grand-finale-orbit-guide/

4. Redd N. How Far Away is Saturn? [Internet]. Space.com. 2020 [cited 7 November 2020]. Available from: https://www.space.com/18477-how-far-away-is-saturn.html

5. Curtis H. Orbital mechanics for engineering students. 3rd ed. Elsevier; 2014.

Appendix (Matlab Code)

```
clear all
close all
clc

%% Units

Cy = 36525;                   % Julian Century (days)
AU = 1.49597871e8;            % Astronomical Unit (km)
arcs = 1/3600;                % Arcsecond in degrees (" in degrees)
muE = 398600.4418;            % Gravitational parameter of Earth (km^3/s^2)
muS =  3.7931187e7;           % Gravitational parameter of Saturn (km^3/s^2)
muS = 1.32712440018e11

%% Defining orbital elements

% Earth

aE = 1.00000261;                % Semi-major axis (AU)
aE_dot = 0.00000562;            % Rate semiaxis is changing per Julian Century
(AU/Cy)
eE = 0.01671123;                % Eccentricity
eE_dot = -0.00004392;           % Change of eccentricity per Julian Century (1/Cy)
iE = -0.00001531;               % Inclination (degrees)
iE_dot = -0.01294668;           % Inclination per Julian Century (¬∫/Cy)
omE = 0;                        % RAAN (degrees)
omE_dot = 0;      % (¬∫/Cy)
wbE = 102.93768193;           % Longitude of perihelion (degrees)
wbE_dot = 0.32327364;         % (¬∫/Cy)
LE = 100.46457166;           % Mean Longitude (degrees)
LE_dot = 35999.37244981;     % (¬∫/Cy)

% Saturn

aS = 9.53667594;
aS_dot = -0.00125060;
eS = 0.05386179;
eS_dot = -0.00050991;
iS = 2.48599187;
iS_dot = 0.00193609;
omS = 113.66242448;
omS_dot = -0.28867794;
wbS = 92.59887831;
wbS_dot = -0.41897216;
LS = 49.95424423;
LS_dot = 1222.49362201;

% Days from j2000 to 15 September 2017

YY = 2017;                    % Year
MM = 9;                       % Month
DD = 15;                      % Day

if MM <= 2
    y = YY-1;
    m = MM+12;
elseif MM > 2
    y = YY;
    m = MM;
end
```

```
J0 = 367*y - fix((7*(y+fix((m+9)/12)))/4) + fix((275*m)/9) + DD + 1721013.5;
UT = -(12-(3 + (31/60)));
JD = J0 + (UT/24);      % Julian day

T0 = (JD - 2451545)/Cy;

% Orbital elements on 23 Apr
% Earth

aE = (aE + aE_dot*T0)*AU;          % 1.496e+8 conversion from astronaumical units
to km
eE = eE + eE_dot*T0;
iE = iE + iE_dot*T0;
omE = omE + omE_dot*T0;
wbE = wbE + wbE_dot*T0;
LE = LE + LE_dot*T0;

if LE > 360
    while LE > 360
        LE = LE - 360;
    end
elseif LE < 0
    while LE < 0
        LE = LE + 360;
    end
end

wE = wbE - omE;                 % Argument of Perihelion (degree)
ME = LE - wbE;                  % Mean anomaly (degree)

% Saturn

aS = (aS + aS_dot*T0)*AU;
eS = eS + eS_dot*T0;
iS = iS + iS_dot*T0;
omS = omS + omS_dot*T0;
wbS = wbS + wbS_dot*T0;
LS = LS + LS_dot*T0;

if LS > 360
    while LS > 360
        LS = LS - 360;
    end
elseif LS < 0
    while LS < 0
        LS = LS + 360;
    end
end

wS = wbS - omS;
MS = LS - wbS;

% Mean anomaly of Earth

if iE < 0

    while iE < 0
        iE = iE+360;
    end
elseif iE > 360

    while iE > 360
        iE = iE - 360;
    end
```

```
end

if ME > 360
    while ME > 360

        ME = ME - 360;

    end
elseif ME < 0
    while ME < 0

        ME = ME + 360;

    end
else

    ME = ME;
end

if wE > 360
    while wE > 360
        wE = wE - 360;
    end
elseif wE < 0
    while wE < 0
        wE = wE + 360;
    end
end

% Mean anomaly of Saturn

if MS > 360
    while MS >360

        MS = MS - 360;

    end
elseif MS < 0
    while MS < 0

        MS = MS + 360;

    end
else

    MS = MS
end

if iS < 0

    while iS < 0
        iS = iS+360;
    end
elseif iS > 360

    while iS > 360
        iS = iS - 360;
    end
end
```

```
if wS > 360
    while wS > 360
        wS = wS - 360;
    end
elseif wS < 0
    while wS < 0
        wS = wS + 360;
    end
end



%% Kepler's equation - Eccentric anomaly (Newton-Raphson Method)

diff = 1e-20;
% Earth

errE = 1;
EE = 270;                      % starting value for eccentric anomaly for Earth

while errE > diff

    EE_old = EE;
    f = EE - eE*sind(EE) - ME;
    df = 1 - eE*cosd(EE);
    EE = EE - f/df;

    errE = abs(EE - EE_old);
end

% Saturn

errS = 1;
ES = MS;

while errS > diff

    ES_old = ES;
    f = ES - eS*sind(ES) - MS;
    df = 1 - eS*cosd(ES);
    ES = ES - f/df;

    errS = abs(ES - ES_old);
end

%% True Anomaly

% Earth

thetaE = 2*atand(sqrt((1+eE)/(1-eE))*tand(EE/2));


if thetaE < 0
    while thetaE < 0
    thetaE = thetaE + 360;
    end
elseif thetaE > 360
    while thetaE > 360
    thetaE = thetaE - 360;
    end
else
    thetaE = thetaE;
end
```

```
% Saturn

thetaS = 2*atand(sqrt((1+eS)/(1-eS))*tand(ES/2));

if thetaS < 0
    while thetaS < 0
    thetaS = thetaS + 360;
    end
elseif thetaS > 360
    while thetaS > 360
        thetaS = thetaS - 360;
    end
else

    thetaS = thetaS;
end

%% Coordinate systems

% Earth
hE = sqrt(muS*aE*(1-eE^2));  % Angular momentum (kg*m^2*s^-1)
RpE = ((hE^2/muS)*(1/(1+eE*cosd(thetaE))))*[cosd(thetaE); sind(thetaE); 0];


% Rotation vector

R_omE = [cosd(omE), -sind(omE), 0; sind(omE), cosd(omE), 0; 0, 0, 1];

R_iE = [1, 0, 0; 0, cosd(iE), -sind(iE); 0, sind(iE), cosd(iE)];

R_wE = [cosd(wE), -sind(wE), 0; sind(wE), cosd(wE), 0; 0, 0, 1];

QE = R_omE*R_iE*R_wE;

RE = QE*RpE;          % Heliocentric coordinates

% Saturn
hS = sqrt(muS*aS*(1-eS^2));
RpS = (hS^2/muS)*(1/(1+eS*cosd(thetaS)))*[cosd(thetaS); sind(thetaS); 0];

% Rotation vector

R_omS = [cosd(omS), -sind(omS), 0; sind(omS), cosd(omS), 0; 0, 0, 1];

R_iS = [1, 0, 0; 0, cosd(iS), -sind(iS); 0, sind(iS), cosd(iS)];

R_wS = [cosd(wS), -sind(wS), 0; sind(wS), cosd(wS), 0; 0, 0, 1];

QS = R_omS*R_iS*R_wS;

RS = QS*RpS;

%% Finding RA and Declination from Cartesian coordinates

% Coordinates of Saturn relative to the Earth

R = RS - RE;

r = sqrt(R(1,1)^2+R(2,1)^2+R(3,1)^2);

% Computing celestial longitude (lam) and celestial latitude (bet)
```

```
R = RS - RE;

r = sqrt(R(1,1)^2+R(2,1)^2+R(3,1)^2);

% Computing celestial longitude (lam) and celestial latitude (bet)

bet = asind(R(3,1)/r);
lam = asind(R(2,1)/(r*cosd(bet)));

% Computing the right accension (delta) and declination (alpha) and epi

epi = 23.5;
delta = asind(sind(bet)*cosd(epi)+cosd(bet)*sind(lam)*sind(epi));
alpha = asind((-
sind(bet)*sind(epi)+cosd(bet)*sind(lam)*cosd(epi))/cosd(delta));

% Converting degree decimal to Hours, minutes and seconds
if alpha < 0

    alpha1 = alpha + 360;
end
alpha1 = alpha1/15;
a_hr = fix(alpha1);
a_min1 = (alpha1 - a_hr)*60;
a_min = fix(a_min1);
a_sec = (a_min1-a_min)*60;
a_sec = fix(a_sec);

% converts degree decimal to degrees, minutes and seconds
d_deg = fix(delta);
d_min1 = (delta-d_deg)*60;
d_min = fix(d_min1);
d_sec = (d_min1-d_min)*60;
d_sec = fix(d_sec);
```

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Escola Superior d'Enginyeries Industrial, Aeroespacial i Audiovisual de Terrassa

ESEIAAT

Astrodynamics

Assignment 1 on Reference Frames

Master's degree in Aerospace Engineering

DQ

November 10, 2020

Contents

List of Figures

1 Statement

In this first assignment, we are asked to compute the right ascension and declination of the Cassini spacecraft from Earth, during the Grand Finale atmospheric entry into Saturn. The assignment is divided in two main parts.

1.1 First part: Heliocentric frame

In the first one we determine the ecliptic coordinates once the heliocentric ecliptic XYZ coordinates of Saturn are computed, given:

$$x = r\cos\beta\cos\lambda \tag{1}$$
$$y = r\cos\beta\sin\lambda \tag{2}$$
$$z = r\sin\beta \tag{3}$$

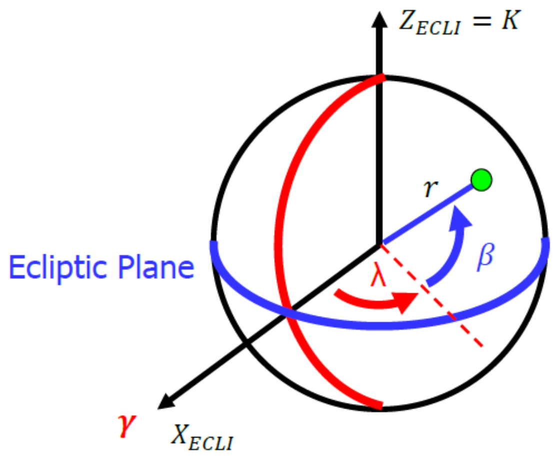


Figure 1: Reference frames

1.2 Second Part: Geocentric frame

In the second activity, we are asked to obtain the declination and the right ascension from the Earth's geocentric frame, solving the spherical triangle shown in Figure 2, composed by:

- Ecliptic Pole (K)
- North Pole (P)
- Saturn position (X)

Using the formulae:

$$\sin \delta = \sin \beta \cos \epsilon + \cos \beta \sin \lambda \sin \epsilon \tag{4}$$

$$\cos \delta \cos \alpha = \cos \beta \cos \lambda \tag{5}$$

$$\cos \delta \sin \alpha = -\sin \beta \sin \epsilon + \cos \beta \sin \lambda \cos \epsilon \tag{6}$$

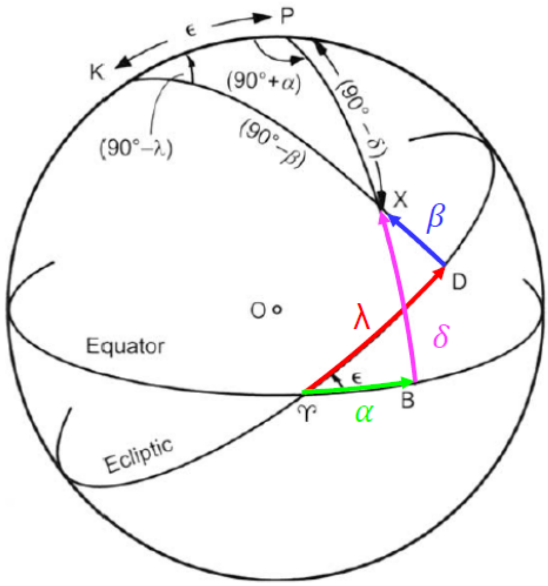


Figure 2: The Spherical Triangle

2 Description of the methodology

The proposed problem will be solved using MATLAB, and the reference data will be obtained from the Jet Propulsion Laboratory's (JPL) Solar System Dynamics database. The main inputs will be the approximate orbital elements of Saturn and Earth, and the desired evaluation time of the event. The procedure is explained in the following section.

2.1 Code structure

The code used to solve the problem is structured in seven different parts, detailed below:

- Definition of the Solar system data
  - Sun's mass $M_{\odot}$ [kg]
  - Universal gravitational constant G [m3/kg/s2]
  - Sun's gravitational standard parameter $\mu_{\odot}$ [m3/s2]
  - Earth's equatorial plane inclination $\epsilon_{\oplus}$ [rad]
- Definition of the evaluation time: Cassini's Grand Finale, real-time downlink initiation.
  - Time chose: September 15 at 12:15 PDT (08:15 UTC)
  - Transform into Julian Calendar
- Definition of the Keplerian Orbital Elements:
  - Definition of the orbital parameters of the planets using JPL's information, for the chosen evaluation time. The data is given as a base Keplerian element value plus a change rate value, referenced to the Julian Century.
    * Semi-major axis a [UA]
    * Eccentricity e
    * Inclination to the ecliptic plane i_{ϵ} [deg]
    * Mean Longitude L [deg]
    * Longitude of perihelion L_{pe} [deg]
    * Longitude of ascending node L_{an} [deg]

    In addition to the given parameters, some others can be obtained directly from the known data.
    * Mean Anomaly M [deg]
    * Argument of perihelion A_{pe} [deg]
- Kepler's Equation: Eccentric anomaly.
  - Kepler's equation is a transcendental equation, meaning it cannot be solved for the eccentric anomaly (E) algebraically. Numerical analysis and series expansions are generally required to evaluate E. In our case, the Newton-Rhapson

method will be used to solve the equation.
The solver is given a limit error (10^{-7}) and an iterative process is executed comparing the difference between the previous and actual value until the result has less error than the desired one. The code also allows to select the bisection method, to solve the equation. It also permits the selection of three different initial values of E:

* $E_0 = M$
* $E_0 = \pi$
* $E_0 = M + e \cdot cos(M)$

- Planet's position vectors.
  - Obtain the true anomaly θ from E.
  - Calculation of the angular momentum h.
  - Calculation of the planet's perifocal radii r.
  - Definition of the 313 $[L_{an} \ i_\epsilon \ A_{pe}]$ rotation matrix.
  - Planet's heliocentric frame position vectors.
  - Planet's relative position vectors.
- Celestial angular longitude λ and latitude β (see Figure 1 and 2).
- Computation of Saturn's Right Ascension α and declination δ in the geocentric frame.

3 Data and results

The different data values given to and obtained from the code are declared as follows:

- Evaluation time: Cassini's Grand Finale.
  - Date: September 15, 12:15 am PDT (08:15 UTC).
  - Julian date: 2458011.84375.
  - Julian century: 0.1770525.
- Keplerian orbital elements and rate of change:

Element	**a**	**e**	**I**	**L**	**Lpe**	**Lan**
Units	AU, AU/Cy	adim, 1/Cy	deg, deg/Cy	deg, deg/Cy	deg, deg/Cy	deg, deg/Cy
Earth (base)	1.00000261	0.01671123	-0.00001531	100.46457166	102.93768193	0.0
Earth (rate)	0.00000562	-0.00004392	-0.01294668	35999.37244981	0.32327364	0.0
Saturn (base)	9.53667594	0.05386179	2.48599187	49.95424423	92.59887831	113.66242448
Saturn (rate)	-0.00125060	-0.00050991	0.00193609	1222.49362201	-0.41897216	-0.28867794

Table 1: Approximate keplerian orbital elements data.

Element	**a**	**e**	**i**	**L**	**Lpe**	**Lan**	**M**	**Ape**
Units	AU	adim	deg	deg	deg	deg	deg	deg
Earth	1.17705560	0.01670345	-0.002307	354.244632	102.994918	0	251.2497143	102.994918
Saturn	11.2251685	0.05377151	2.4863346	266.399836	92.5246982	113.611313	173.8751377	-21.0866151

Table 2: Evaluated orbital elements.

- Eccentric and true anomalies, angular momentum.

	E	θ	**h**
Units	rad	rad	Mixed SI + AU
Earth	4.36940413	-1.92946844	$3.85461287\text{x}10^{1}0$
Saturn	3.04013951	3.04544731	$1.24983013\text{x}10^{1}0$

Table 3: Eccentric, true anomalies and angular momentum.

- Position vectors.
  - Perifocal radii:
    * $r_{\oplus} = 1.183667$ AU
    * $r_{♄} = 11.825659$ AU
  - Heliocentric position vectors:
    * $ECLI_{\oplus} = [1.173390961;\ -0.155635898;\ 0.000006268]$ AU
    * $ECLI_{♄} = [-0.611047636;\ -11.807628392;\ 0.229667650]$ AU
  - Relative position:
    * $ECLI_{♄-\oplus} = [-1.784438597;\ -11.651992494;\ 0.229661382]$ AU
    * $r_{♄-\oplus} = 11.790076$ AU
- Celestial angular position
  - $\beta_{♄,\oplus} = 1.116147$ deg
  - $\lambda_{♄,\oplus} = 261.293113$ deg
- **Geocentric angular position**
  - **Right ascension** $\alpha_{♄,\oplus} = 260.602687$ **deg**
  - **Declination** $\delta_{♄,\oplus} = -22.036882$ **deg**

4 Conclusions

It is possible to check a multitude of different parameters in the HORIZONS web interface of JPL's Solar System Dynamics directory. That way, we can confirm if the obtained results are correct or differ from the actual ephemerides. The input parameters used in the interface are, as stated in the web:

Ephemeris Type: OBSERVER
Target Body: Saturn [699]
Observer Location: Geocentric [500]
Time Span: Start=2017-09-15 08:10, Stop=2017-09-15 08:20, Step=1 m
Table Settings: QUANTITIES=1,2,18,28,41
Display/Output: default (formatted HTML)

The shown Right Ascension and Declination are:

- Saturn R.A.: 17h 23min 26.69s = 260.8612 deg
- Saturn Dec.: -22º 03min 11.0s = -22.05306 deg

As we can see, the error is less than 0.1% in both cases. It should be stated that the orbital elements used in our calculations are an approximation, albeit one that gives excellent results, at least when used within JPL's stated time-span for these values, from 1800 AD to 2050 AD.

A MATLAB code

```

clear all
close all
format long
clc

%% Solar system data
Ms = 1.989e30;            % Sun's mass [kg]
G = 6.67408e-11;          % Gravitational constat [m3/kg/s2]
mu_s = Ms*G;              % Sun's gravitational standar parameter [m3/s2]
ie = deg2rad(23.43658); % Equatorial plane inclination [rad]
AU = 149597870700;        % [m]

%% Evaluation time (Grand Finale)
% Time when Real-Time downlink is initiated
% September 15, 12:15 am PDT
% September 15, 08:15 UTC

year  = 2017;
month = 9;
day   = 15;
hour  = 8;
min   = 15;
sec   = 0;

% To Julian calendar
t_string =  num2str(year) + "-" + num2str(month) + "-" + num2str(day)+ ...
    " "+ ...
            num2str(hour) + ":" + num2str(min) + ":" + num2str(sec);

t = datetime(t_string);

JD = juliandate(t);

JC = ( JD - 2451545.0 ) / 36525;

%% Orbital elements
keplerianElements;

% Semi-major axis [UA]
orbSat.a = Sat.a + Sat.a * JC;
orbEarth.a = Earth.a + Earth.a * JC;

% Eccentricity
orbSat.e = Sat.e + Sat.eR * JC;
orbEarth.e = Earth.e + Earth.eR * JC;

% Inclination to the ecliptic plane [deg]
orbSat.i = Sat.i + Sat.iR * JC;
orbEarth.i = Earth.i + Earth.iR * JC;

```

```
% Mean longitude [deg]
orbSat.L = Sat.L + Sat.LR * JC;
orbEarth.L = Earth.L + Earth.LR * JC;
orbEarth.L = mod(orbEarth.L,360);

% Longitude of perihelion [deg]
orbSat.lp = Sat.lp + Sat.lpR * JC;
orbEarth.lp = Earth.lp + Earth.lpR * JC;

% Longitude of the ascending node [deg]
orbSat.lan = Sat.lan + Sat.lanR * JC;
orbEarth.lan = Earth.lan + Earth.lanR * JC;

% Mean anomaly [deg]
orbSat.M = orbSat.L - orbSat.lp;
orbEarth.M = orbEarth.L - orbEarth.lp;
orbEarth.M = mod(orbEarth.M,360);

% Argument of perihelion [deg]
orbSat.ap = orbSat.lp - orbSat.lan;
orbEarth.ap = orbEarth.lp - orbEarth.lan;


%% Kepler's Equation: Eccentric anomaly

% Solving Kepler's Equation [rad] (Assignment 2)

% Solver: 1 -> Newton Raphson
%         2 -> Bisection
% Initial condition: 1 -> E0 = M
%                    2 -> E0 = 180
%                    3 -> E0 = M+e*cos(M)

init = 3;
solver = 2;

if init == 1
    E0_S = orbSat.M;
    E0_E = orbEarth.M;
elseif init == 2
    E0_S = 180;
    E0_E = 180;
elseif init == 3
    E0_S = orbSat.M + orbSat.e*cosd(orbSat.M);
    E0_E = orbEarth.M + orbEarth.e*cosd(orbEarth.M);
end

if solver == 1
    [S_E,kS,tfS] = keplerNRSolve(orbSat.e,orbSat.M,180);
    [E_E,kE,tfE] = keplerNRSolve(orbEarth.e,orbEarth.M,180);
elseif solver == 2
    diff = 0.5;
    [S_E,kS,tfS] = ...
        keplerBisection(orbSat.e,orbSat.M,(1-diff)*E0_S,(1+diff)*E0_S);
```

```
    [E_E,kE,tfE] = ...
        keplerBisection(orbEarth.e,orbEarth.M,(1-diff)*E0_E,(1+diff)*E0_E);
end

% True anomaly
angS = tan(S_E/2) / sqrt((1-orbSat.e)/(1+orbSat.e));
angE = tan(E_E/2) / sqrt((1-orbEarth.e)/(1+orbEarth.e));

Sat_theta = 2*atan(angS);
Earth_theta = 2*atan(angE);

% Angular momentum
hS = sqrt(mu_s*orbSat.a*(1-orbSat.e^2));
hE = sqrt(mu_s*orbEarth.a*(1-orbEarth.e^2));

%% Position vectors

% Rotation matrix
rotSat = rotz(orbSat.lan)*rotx(orbSat.i)*rotz(orbSat.ap);
rotEarth = rotz(orbEarth.lan)*rotx(orbEarth.i)*rotz(orbEarth.ap);

% Instantaneous radii
rSat_mod = hS^2/mu_s/(1+orbSat.e*cos(Sat_theta));
rEarth_mod = hE^2/mu_s/(1+orbEarth.e*cos(Earth_theta));

rSat = rotSat * rSat_mod*[cos(Sat_theta) sin(Sat_theta) 0]';
rEarth = rotEarth * rEarth_mod*[cos(Earth_theta) sin(Earth_theta) 0]';

% Instantaneous velocity
vSat = rotSat * mu_s/hS*[-sin(Sat_theta) orbSat.e+cos(Sat_theta) 0]';
vEarth = rotEarth * mu_s/hE*[-sin(Earth_theta) ...
    orbEarth.e+cos(Earth_theta) 0]';

% Relative position
rSEv = rSat - rEarth;
rSE = norm(rSEv);


%% Angular position

beta = asind(rSEv(3)/rSE);

% Quadran check
if rSEv(3)<0
    beta = 360 + beta;
end

lambda = asind(rSEv(2)/(rSE*cosd(beta)));

% Quadran check
if rSEv(1)<0
    lambda = 180 - lambda;
end


```

```
%% Relative Right Ascension and Declination

Δ = asind(sind(beta)*cos(ie) + cosd(beta)*sind(lambda)*sin(ie));

num = cosd(beta)*cosd(lambda);
den = cosd(Δ);
raan = acosd(num/den);
raan = 360 - raan;

Δ
raan
```

```

%% Keplerian orbital elements

%                a              e                I                L ...
                long.peri.      long.node.
%            AU, AU/Cy      rad, rad/Cy      deg, deg/Cy       deg, deg/Cy ...
          deg, deg/Cy      deg, deg/Cy
%-----------------------------------------------------------------------

Sat.a     = 9.53667594;
Sat.aR    = -0.00125060;
Sat.e     = 0.05386179;
Sat.eR    = -0.00050991;
Sat.i     = 2.48599187;
Sat.iR    = 0.00193609;
Sat.L     = 49.95424423;
Sat.LR    = 1222.49362201;
Sat.lp    = 92.59887831;
Sat.lpR   = -0.41897216;
Sat.lan   = 113.66242448;
Sat.lanR = -0.28867794;

Earth.a     = 1.00000261;
Earth.aR    = 0.00000562;
Earth.e     = 0.01671123;
Earth.eR    = -0.00004392;
Earth.i     = -0.00001531;
Earth.iR    = -0.01294668;
Earth.L     = 100.46457166;
Earth.LR    = 35999.37244981;
Earth.lp    = 102.93768193;
Earth.lpR   = 0.32327364;
Earth.lan   = 0.0;
Earth.lanR = 0.0;
```

```

function [u,v,w] = velPQW(n,r,E)

    % Velocities in km/s
    % Angular velocity in rev/day
```

```
    % Distances in km

    nrad = n*2*pi/(24*3600);     % [rad/s]

    u = -r*nrad*sind(E);
    v = r*nrad*cosd(E);
    w = 0;

end
```

```

function [E,k,tf] = keplerNRSolve(e,M,E0)

% INPUTS:
% e  = eccentricity [adim]
% M  = mean anomaly [deg]
% E0 = initial condition [deg]
%
% OUTPUTS:
% E  = eccentric anomaly [rad]
% k  = number of iterations
% tf = solving time [s]

M = deg2rad(M);
E0 = deg2rad(E0);

err = 10e-7;  % Error
diff = 1;     % Difference (f/f')
k = 1;        % Iteration counter
E = E0;       % Eccentric anomaly init.

tic
while (abs(diff) > err)

    Fun  = E - e*sin(E) - M;     % Mean anomaly vs Eccentric anomaly
    dFun = 1 - e*cos(E);         % Derivative

    if(abs(dFun) < err)
        break;
    else
        diff = Fun/dFun;
        E = E - diff;
        k = k+1;
    end
end

tf = toc;


end
```

```

```

```
function [Ep,k,tf] = keplerBisection(e,M,Ea,Eb)

% INPUTS:
% e  = eccentricity [adim]
% M  = mean anomaly [deg]
% Ea = Lower boundary E [deg]
% Eb = Upper boundary E [deg]
%
% OUTPUTS:
% Ep = eccentric anomaly [rad]
% k  = number of iterations
% tf = solving time [s]

Ea = deg2rad(Ea);
Eb = deg2rad(Eb);
M = deg2rad(M);

f = @(E) (E - e*sin(E) - M);
k = 1;

tic
if f(Ea)*f(Eb)>0
    disp('Not valid initial guesses')
else
    Ep = (Ea + Eb)/2;
    err = abs(f(Ep));
    while err > 1e-7
        if f(Ea)*f(Ep)<0
            Eb = Ep;
        else
            Ea = Ep;
        end
        Ep = (Ea + Eb)/2;
        err = abs(f(Ep));
        k = k+1;
    end
end

tf = toc;

end
```

Astrodynamics ESEIAAT UPC January 2021
Name: Final Exam
DNI or Passport:

Instructions for all attendees:

There is **ONLY** one correct answer on each question.
Every correctly answered question adds +1.0 point.
Every incorrectly answered question discounts -0.25 points
30 minutes of maximum time to complete the test.
No lecture slides are allowed or Internet resources.

Additional Instructions for remote attendees:

Highlight in **bold** the correct answer.
After completion, **upload the exam to Atenea.**
The camera and microphone must be switched on during the entire exam duration.
Only the mouse can be used during the exam to make the text bold.

0) This is a question sample on how to return the exam.
   a) This answer is NOT selected.
   b) This answer is NOT selected.
   c) This answer is selected and this is why it is bolded.
   d) None of the above

1) What is the central assumption in the patched conic method?
   a) To have multiple sphere of influences acting simultaneously
   b) To have only one central body acting on a given time
   c) To have propulsion acting during the cruise phase
   d) None of the above

2) What is a stable time reference, suitable for timekeeping during an entire space mission?
   a) A timescale based on the daily rotation of the Earth
   b) A timescale based on the yearly translation of the Earth around the sun
   c) A timescale based on an atomic oscillator
   d) None of the above

3) The inclination of the orbit (assuming a direct insertion without manoeuvring):
   a) Is lower or equal than the latitude of the launch site
   b) Is greater or equal than the latitude of the launch site
   c) Depends on the launch time
   d) None of the above

4) In the two body problem, which of the Keplerian elements are not constant?
   a) True Anomaly, Eccentric Anomaly
   b) Semi-major axis, eccentricity
   c) Inclination, longitude of the ascending node
   d) None of the above

5) Which statement is true before and after a gravity assist manoeuvre:
    a) The velocity of the spacecraft relative to the planet is maintained constant
    b) The velocity of the planet relative to the Sun is maintained constant
    c) The velocity of the spacecraft relative to the Sun is maintained constant
    d) None of the above

6) Changing the inclination of the orbital plane is more fuel-efficient:
    a) In the line of nodes, where the spacecraft intersects the plane of reference
    b) At the periapsis, where the velocity is greatest
    c) At the apoapsis, where the velocity is lowest
    d) None of the above

7) The quickest orbital transfer, adequate to respond a medical emergency on board is:
    a) Hyperbolic transfer with chemical propulsion
    b) Hohmann transfer with chemical propulsion
    c) Multiple Hohmann segment with multiple chemical low thrust impulses
    d) Spiral transfer with continuous electrical propulsion

8) In a restricted circular three body problem, the libration point L3 point is located:
    a) Between the principal mass and the secondary mass, close to the secondary mass
    b) In the line between the primary mass and secondary mass, opposite to the secondary mass
    c) +60 degrees from the centre of mass, advanced of the secondary mass
    d) -60 degrees from the centre of mass, retarded of the secondary mass

9) The ascending order of the energy of a Keplerian orbit is:
    a) Circle, Ellipse, Parabola, Hyperbola
    b) Hyperbola, Parabola, Ellipse, Circle
    c) Ellipse, Circle, Hyperbola, Parabola
    d) None of the above

10) In a Keplerian orbit representation, the ascending node is:
    a) The line intersecting the orbit plane and plane of reference (e.g. equator)
    b) The cross product of the position and velocity vectors
    c) The angle between the orbit plane and a plane of reference (e.g. equator)
    d) None of the above

11) The rotation of Earth presents two clear perturbation periods:
    a) Nutation, with a period of 25,765 years and Precession, with a period of 18.6 years
    b) Polar motion with a period of 28 days and sidereal rotation of 23h56 minutes
    c) Precession, with a period of 25,765 years and Nutation, with a period of 18.6 years
    d) Sidereal rotation, with a period of 28 days and Polar motion with a period of 23h56 minutes

12) The effect of the Drag on the orbit
    a) Increase of eccentricity, decrease of perigee height
    b) Decrease of eccentricity, decrease of perigee height
    c) Decrease of eccentricity, decrease of apogee height
    d) Increase of eccentricity, decrease of apogee height

13) The period of the Geosynchronous Equatorial Orbit (GEO) is
   - **a) A sidereal day (i.e. 23 h 56 m 4.09 s)**
   - b) A solar day (i.e. 24 h)
   - c) Proportional to the synodic period
   - d) None of the above

14) Molniya and Tundra Orbits share
   - a) The semi-major axis
   - b) The orbital period
   - **c) The inclination**
   - d) The eccentricity

15) The satellite Ground track on a non-rotating spherical Earth
   - a) It is determined by the intersection of a plane passing through the center of the Earth.
   - b) Corresponds to a great circle
   - c) Repeats the same ground track over and over
   - **d) All of the above**

16) A constellation of satellites is
   - a) Is composed by satellites of very different design
   - **b) Has a common objective (e.g. communication, navigation, science).**
   - c) Is composed by satellites in different orbit each
   - d) All of the above

17) The Lambert theorem states that the transfer time of a body moving between two points on a conic trajectory
   - **a) depends on the chord joining these two positions**
   - b) is independent of the direction of motion
   - c) depends on the conic eccentricity joining these two positions
   - d) None of the above

18) When solving the Lambert problem by Simo's method, the geometric meaning of $\sqrt{z}$ corresponds to
   - a) The eccentricity of the solution orbit
   - **b) half of the variation in eccentric anomaly between $\boldsymbol{P_1}$ and $\boldsymbol{P_2}$ on the solution orbit**
   - c) half of the variation in true anomaly between P_1 and P_2 on the solution orbit
   - d) None of the above

19) Which sentence is not true about the Pork Chop Plot?
   - a) Depicts the results **(e.g.** $\Delta\mathrm{V}, \Delta\theta$) for various combinations of launch time and time-of-flight Δt
   - b) Depend on launch constraints such as the range of allowable launch azimuths
   - c) Provide a preliminary estimate of the amount of propellant to be carried onboard the spacecraft.
   - **d) Every pair of launch time and time-of-flight Δt outputs a different result (e.g.** $\Delta\mathrm{V}, \Delta\theta$) **in the PCP, being no repeated values in the PCP.**

20) The Broken Plane Manoeuvre is a consequence of

- **a) Planetary orbits (departure, arrival) are not co-planar**
- b) Planetary orbits (departure, arrival) are eccentric
- c) The low ΔV requirements of polar transfers
- d) None of the above

21) What is the use of the Tisserand Graph ?

- a) To obtain the synodic periods in interplanetary missions
- b) To determine whether a mass (e.g. a comet) is the same before/after a gravity assist maneuver
- c) **To preliminary design interplanetary missions by means of gravity assists**
- d) None of the above

22) In the restricted three-body problem, the third mass (e.g. a spacecraft)

- a) Can move freely in any position with any velocity
- **b) Has forbidden regions that cannot cross into, depending on the value of the Jacobi constant**
- c) The velocity is zero for any value of the Jacobi constant.
- d) None of the above

23) In the n-body problem,

- a) The Total Linear Momentum is conserved, as in the two-body problem
- b) The Total Angular Momentum is conserved, as in the two-body problem
- c) The Total Energy of the system is conserved, as in the two-body problem
- **d) All of the above**

24) The Sphere of Influence SOI:

- a) Inside the planet SOI, the motion of the Space Vehicle is determined by its equations of motion relative to the Sun
- **b) Its size is proportional to the mass of the planet versus the Sun**
- c) Its size is proportional to the distance from the planet to the Sun
- d) None of the above

25) Numerical Iterative methods

- a) Always convergence to a stable solution with an adequate numerical precision
- **b) Can require derivatives of first, second or third order**
- c) The initialization guess must be good enough to assure numerical convergence
- d) None of the above

Question 1 (4 points):

Compute the ΔV and necessary time to reduce by half the orbital period of a synchronous satellite with Neptune rotation using an inward Hohman transfer.

Assuming

$$\mu_N = 6.836529 \cdot 10^{15}\ m^3/s^2$$

$$R_N = 24764\ km$$

a) Assuming an orbit inclination of cero degrees and a circular orbit, determine the orbit height at which the orbiter would be synchronised with the Neptune's sidereal period of $16.11\ h$

$$\mathrm{T} = 2\pi\sqrt{\frac{a^3}{\mu}}$$

$$a = \sqrt[3]{\mu_N\left(\frac{T}{2\pi}\right)^2} = \sqrt[3]{6.836529 \cdot 10^{15} \cdot \left(\frac{16.11 \cdot 3600}{2\pi}\right)^2} = 83513625.04\ m$$

$$h = a - R_N = 83514 \cdot 10^3 - 24764 \cdot 10^3 = 58749625.04\ m = 58749.63\ km$$

b) Determine the final semi-major axis of the final semi-synchronous orbit

$$\mathrm{T} = 2\pi\sqrt{\frac{a^3}{\mu}}$$

$$a = \sqrt[3]{\mu_N\left(\frac{T/2}{2\pi}\right)^2} = \sqrt[3]{6.836529 \cdot 10^{15} \cdot \left(\frac{16.11 \cdot 3600}{4\pi}\right)^2} = 52610287.07\ m$$

c) For the Hohman transfer, compute the eccentricity and the time required to perform the manoeuvre:

$$e_{trans} = \frac{r_0 - r_f}{r_0 + r_f} = \frac{83513625.04 - 52610287.07}{83513625.04 + 52610287.07} = 0.227$$

$$a_{trans} = \frac{r_0 + r_f}{2} = \frac{83513625.04 + 52610287.07}{2} = 68061956.05\ m$$

$$\Delta t_{trans} = \frac{T_{trans}}{2} = \pi\sqrt{\frac{a_{trans}^3}{\mu}} = \pi\sqrt{\frac{68061956.05^3}{6.836529 \cdot 10^{15}}} = 21334.81\ s = 355.58\ min = 5.9263\ h$$

d) Determine the initial, final and total required ΔV, plotting (sketching) where each ΔV takes places, and its direction:

$$V_0 = \sqrt{\frac{\mu}{r_0}} = \sqrt{\frac{6.836529 \cdot 10^{15}}{83513625.04}} = 9047.72\ m/s$$

$$V_a = \sqrt{\frac{2 \cdot \mu}{r_0} - \frac{\mu}{a_{trans}}} = \sqrt{\frac{2 \cdot 6.836529 \cdot 10^{15}}{83513625.04} - \frac{6.836529 \cdot 10^{15}}{68061956.05}} = 7954.67\ m/s$$

$$V_p = \sqrt{\frac{2 \cdot \mu}{r_f} - \frac{\mu}{a_{trans}}} = \sqrt{\frac{2 \cdot 6.836529 \cdot 10^{15}}{52610287.07} - \frac{6.836529 \cdot 10^{15}}{68061956.05}} = 12627.25\ m/s =$$

$$V_f = \sqrt{\frac{\mu}{r_f}} = \sqrt{\frac{6.836529 \cdot 10^{15}}{52610287.07}} = 11399.41\ m/s$$

$$\Delta V_1 = V_a - V_0 = 7954.67 - 9047.72 = -1093.05\ m/s$$

$$\Delta V_2 = V_p - V_a = 11399.41 - 12627.25 = -1227.84\ m/s$$

$$\Delta V_T = |\Delta V_1| + |\Delta V_2| = 1093.05 + 1227.84 = 2320.89\ m/s$$

e) Assuming a solid chemical propulsion system of $\mathrm{I_{sp}} = 300$ s, compute the required mass fraction of propellant to perform such maneuver

$$\frac{\Delta m}{m_0} = 1 - e^{-\frac{\Delta V}{I_{sp} \cdot g_{SL}}} = 1 - e^{-\frac{2320.89}{300 \cdot 9.81}} = 0.5492$$

Question 2 (2 points):

A geocentric parabolic orbit has a perigee radius of 6600 km

Assuming

$$R_E = 6378 \text{ km}$$
$$\mu_E = 3.986 \cdot 10^{14} \ m^3/s^2$$

a) Determine the flight time from $\theta = -90^\circ$ to $\theta = +90^\circ$

The orbit equation, particularized at the perigee yields the angular momentum vector:

$$r = \frac{h^2}{\mu} \frac{1}{1 + \cos\theta}$$

$$r_p = \frac{h^2}{\mu_E} \frac{1}{1 + \cos 0}$$

$$h = \sqrt{2 \cdot r_p \cdot \mu_E} = \sqrt{2 \cdot 6600 \cdot 10^3 \cdot 3.986 \cdot 10^{14}} = 72536335722.17 \, m^3/s^2$$

The Barker's equation, particularized at each true anomaly

$$M_{\theta=90} = \frac{1}{2}\tan\left(\frac{90}{2}\right) + \frac{1}{6}\tan^3\left(\frac{90}{2}\right) = \frac{1}{2} + \frac{1}{6} = \frac{2}{3} rad$$

$$M_{\theta=-90} = \frac{1}{2}\tan\left(\frac{-90}{2}\right) + \frac{1}{6}\tan^3\left(\frac{-90}{2}\right) = -\frac{1}{2} - \frac{1}{6} = -\frac{2}{3} \ rad$$

Then

$$\frac{\mu^2}{h^3}(t_2 - t_1) = M_2 - M_1$$

$$\Delta t = \frac{M_2 - M_1}{\frac{\mu^2}{h^3}} = \frac{\frac{2}{3} - (-\frac{2}{3})}{\frac{(3.986 \cdot 10^{14})^2}{(72536335722.17)^3}} = 3202.80 \ s = 53.38 \ min = 0.88967 \ h$$

b) Determine the geocentric vector (r, θ) after 24 h of orbiting the perigee.

We known that the parabolic mean anomaly can be directly computed by the time since the periapsis:

$$\frac{\mu^2}{h^3} t = M$$

$$M = \frac{{\mu_E}^2}{h^3} t = \frac{(3.986 \cdot 10^{14})^2}{(72536335722.17)^3} 24 \cdot 3600 = 35.96 \text{ rad} = 2060.84^\circ$$

That can be related to the true anomaly by means of:

$$\tan\left(\frac{\theta}{2}\right) = \left(3M + \sqrt{(3\,M)^2 + 1}\right)^{\frac{1}{3}} - \left(3M + \sqrt{(3\,M)^2 + 1}\right)^{-\frac{1}{3}}$$

$$\tan\left(\frac{\theta}{2}\right) = \left(3 \cdot 35.96 + \sqrt{(3\ \cdot 35.96)^2 + 1}\right)^{\frac{1}{3}} - \left(3 \cdot 35.96 + \sqrt{(3\ \cdot 35.96)^2 + 1}\right)^{-\frac{1}{3}}$$

$$\tan\left(\frac{\theta}{2}\right) = 5.83\, rad$$

$$\theta = 160.54^\circ$$

Hence, we can obtain the distance from the geocenter:

$$r = \frac{h^2}{\mu} \frac{1}{1 + \cos\theta} = \frac{72536335722.17^2}{3.986 \cdot 10^{14}} \frac{1}{1 + \cos 160.54} = 231047912.72\, m$$

+5 -4

Astrodynamics ESEIAAT UPC January 2021

Name: LN Final Exam

DNI or Passport:

Instructions for all attendees:

This theoretical part weights 4 points out of 10 in the final exam
There is **ONLY** one correct answer on each question.
Every correctly answered question adds +1.0 point.
Every incorrectly answered question discounts -0.25 points
30 minutes of maximum time to complete the test.
No lecture slides are allowed or Internet resources.

Additional Instructions for remote attendees:
Highlight in **bold** the correct answer.
After completion, **email-me the exam.**
The camera and microphone must be switched on during the entire exam duration.
Only the mouse can be used during the exam to make the text bold.

0) This is a question sample on how to return the exam for remote atendees.
   a) This answer is NOT selected.
   b) This answer is NOT selected.
   c) This answer is selected and this is why it is bolded.
   d) None of the above

1) What is the central assumption in the patched conic method?
   a) To have multiple sphere of influences acting simultaneously
   b) To have only one central body acting on a given time
   c) To have propulsion acting during the cruise phase
   d) None of the above

2) What is a stable time reference, suitable for timekeeping during an entire space mission?
   a) A timescale based on the daily rotation of the Earth
   b) A timescale based on the yearly translation of the Earth around the sun
   c) A timescale based on an atomic oscillator
   d) None of the above

3) The inclination of the orbit (assuming a direct insertion without manoeuvring):
   a) Is lower or equal than the latitude of the launch site
   b) Is greater or equal than the latitude of the launch site
   c) Depends on the launch time
   d) None of the above

4) In the two body problem, which of the Keplerian elements are not constant?
   a) True Anomaly, Eccentric Anomaly
   b) Semi-major axis, eccentricity
   c) Inclination, longitude of the ascending node
   d) None of the above

5) Which statement is true before and after a gravity assist manoeuvre:
   a) The velocity of the spacecraft relative to the planet is maintained constant
   b) The velocity of the planet relative to the Sun is maintained constant -1
   c) The velocity of the spacecraft relative to the Sun is maintained constant
   d) None of the above

6) Changing the inclination of the orbital plane is more fuel-efficient:
   a) In the line of nodes, where the spacecraft intersects the plane of reference -1
   b) At the periapsis, where the velocity is greatest
   c) At the apoapsis, where the velocity is lowest
   d) None of the above

7) The quickest orbital transfer, adequate to respond a medical emergency on board is:
   a) Hyperbolic transfer with chemical propulsion
   b) Hohmann transfer with chemical propulsion
   c) Multiple Hohmann segment with multiple chemical low thrust impulses
   d) Spiral transfer with continuous electrical propulsion

8) In a restricted circular three body problem, the libration point L3 point is located:
   a) Between the principal mass and the secondary mass, close to the secondary mass
   b) In the line between the primary mass and secondary mass, opposite to the secondary mass
   c) +60 degrees from the centre of mass, advanced of the secondary mass
   d) -60 degrees from the centre of mass, retarded of the secondary mass

9) The ascending order of the energy of a Keplerian orbit is:
   a) Circle, Ellipse, Parabola, Hyperbola
   b) Hyperbola, Parabola, Ellipse, Circle 1
   c) Ellipse, Circle, Hyperbola, Parabola
   d) None of the above

10) In a Keplerian orbit representation, the ascending node is:
   a) The line intersecting the orbit plane and plane of reference (e.g. equator)
   b) The cross product of the position and velocity vectors
   c) The angle between the orbit plane and a plane of reference (e.g. equator)
   d) None of the above

11) The rotation of Earth presents two clear perturbation periods:
   a) Nutation, with a period of 25,765 years and Precession, with a period of 18.6 years
   b) Polar motion with a period of 28 days and sidereal rotation of 23h56 minutes
   c) Precession, with a period of 25,765 years and Nutation, with a period of 18.6 years
   d) Sidereal rotation, with a period of 28 days and Polar motion with a period of 23h56 minutes

12) The effect of the Drag on the orbit
   a) Increase of eccentricity, decrease of perigee height
   b) Decrease of eccentricity, decrease of perigee height 1
   c) Decrease of eccentricity, decrease of apogee height
   d) Increase of eccentricity, decrease of apogee height

13) The period of the Geosynchronous Equatorial Orbit (GEO) is

a) A sidereal day (i.e. 23 h 56 m 4.09 s)
b) A solar day (i.e. 24 h)
c) Proportional to the synodic period
d) None of the above

14) Molniya and Tundra Orbits share

a) The semi-major axis
b) The orbital period
c) The inclination
d) The eccentricity

15) The satellite Ground track on a non-rotating spherical Earth

a) It is determined by the intersection of a plane passing through the center of the Earth.
b) Corresponds to a great circle
c) Repeats the same ground track over and over
d) All of the above

16) A constellation of satellites is

a) Is composed by satellites of very different design
b) Has a common objective (e.g. communication, navigation, science). 1
c) Is composed by satellites in different orbit each
d) All of the above

17) The Lambert theorem states that the transfer time of a body moving between two points on a conic trajectory

a) depends on the chord joining these two positions 1
b) is independent of the direction of motion
c) depends on the conic eccentricity joining these two positions
d) None of the above

18) When solving the Lambert problem by Simo's method, the geometric meaning of $\sqrt{z}$ corresponds to

a) The eccentricity of the solution orbit
b) half of the variation in eccentric anomaly between P_1 and P_2 on the solution orbit
c) half of the variation in true anomaly between P_1 and P_2 on the solution orbit
d) None of the above

19) Which sentence is not true about the Pork Chop Plot?

a) Depicts the results (e.g. $\Delta V, \Delta\theta$) for various combinations of launch time and time-of-flight Δt
b) Depend on launch constraints such as the range of allowable launch azimuths
c) Provide a preliminary estimate of the amount of propellant to be carried onboard the spacecraft.
d) Every pair of launch time and time-of-flight Δt outputs a different result (e.g. $\Delta V, \Delta\theta$) in the PCP, being no repeated values in the PCP. -1

20) The Broken Plane Manoeuvre is a consequence of
   a) Planetary orbits (departure, arrival) are not co-planar
   b) Planetary orbits (departure, arrival) are eccentric
   c) The low ΔV requirements of polar transfers
   d) None of the above

21) What is the use of the Tisserand Graph ?
   a) To obtain the synodic periods in interplanetary missions
   b) To determine whether a mass (e.g. a comet) is the same before/after a gravity assist maneuver
   c) To preliminary design interplanetary missions by means of gravity assists
   d) None of the above

22) In the restricted three-body problem, the third mass (e.g. a spacecraft)
   a) Can move freely in any position with any velocity
   b) Has forbidden regions that cannot cross into, depending on the value of the Jacobi constant
   c) The velocity is zero for any value of the Jacobi constant.
   d) None of the above

23) In the n-body problem,
   a) The Total Linear Momentum is conserved, as in the two-body problem
   b) The Total Angular Momentum is conserved, as in the two-body problem
   c) The Total Energy of the system is conserved, as in the two-body problem
   d) All of the above

24) The Sphere of Influence SOI:
   a) Inside the planet SOI, the motion of the Space Vehicle is determined by its equations of motion relative to the Sun
   b) Its size is proportional to the mass of the planet versus the Sun
   c) Its size is proportional to the distance from the planet to the Sun
   d) None of the above

25) Numerical Iterative methods
   a) Always convergence to a stable solution with an adequate numerical precision
   b) Can require derivatives of first, second or third order
   c) The initialization guess must be good enough to assure numerical convergence
   d) None of the above

0.5 + 2 = 2.5

Astrodynamics ESEIAAT UPC
Name: LN
DNI or Passport:

January 2021
Final Exam

Question 1 (4 points):

Compute the ΔV and necessary time to reduce by half the orbital period of a synchronous satellite with Neptune rotation using an inward Hohman transfer.

Assuming

$$\mu_N = 6.836529 \cdot 10^{15}\ m^3/s^2$$

$$R_N = 24764\ km$$

a) Assuming an orbit inclination of cero degrees and a circular orbit, determine the orbit height at which the orbiter would be synchronised with the Neptune's sidereal period of $16.11\ h$
b) Determine the final semi-major axis of the final semi-synchronous orbit ($i.e.\ \mathrm{T_f} = \mathrm{T_0}/2$)
c) For the Hohman transfer, compute the eccentricity and the time required to perform the manoeuvre:
d) Determine the initial, final and total required ΔV, plotting (sketching) where each ΔV takes places, and its direction:
e) Assuming a solid chemical propulsion system of $\mathrm{I_{sp}} = 300$ s, compute the required mass fraction of propellant to perform such maneuver.

Question 2 (2 points):

A geocentric parabolic orbit has a perigee radius of 6600 km

Assuming

$$\mathrm{R_E} = 6378\ \mathrm{km}$$
$$\mu_\mathrm{E} = 3.986 \cdot 10^{14}\ m^3/s^2$$

a) Determine the flight time from $\theta = -90^\circ$ to $\theta = +90^\circ$
b) Determine the geocentric vector (r, θ) after 24 h of orbiting the perigee.

FINAL ASTRODYNAMICS

1. ΔV ? $T/2$

$\mu_N = 6{,}836529 \cdot 10^{15}\ m^3/s^2$
$R_N = 24764$ km

} Hohmann Transfer inward

a) orbit inc = 0° and circular orbit

$T = 16{,}11\ h = 57996\ s \longrightarrow T = 2\pi\sqrt{\frac{a^3}{\mu}} \longrightarrow a = \sqrt[3]{\mu\frac{T^2}{4\pi^2}} = \sqrt[3]{\mu_N \cdot \frac{57996^2}{4\pi^2}}$

$a_T = 83{,}5136 \cdot 10^6\ m$

0.5

$r_{parking} = R_N + h = 24764 \cdot 10^3 + h$

$a_{trans} = \frac{r_0 + r_f}{2} \longrightarrow a_{trans} = \frac{R_N + R_N + h}{2} \longrightarrow h = 2a_{trans} - 2R_N$

$h = 117{,}4992 \cdot 10^6\ m \longrightarrow \boxed{h = 117499{,}2\ km}$

b) final semi major axis of $T_0/2$

$|a| = \frac{\mu_N}{2V_{\infty,N}^2} \longrightarrow V_{\infty N} = \Delta V_1 = \sqrt{\frac{2\mu_N}{r_0} - \frac{\mu_N}{a_{trans}}} - \sqrt{\frac{\mu_N}{r_0}}$

if $T_f = T_0/2$ then a_{trans}

$a_T = \sqrt[3]{\mu_N\frac{(T/2)^2}{4\pi^2}} = 52{,}61 \cdot 10^6\ m$

$V_{\infty N} = \sqrt{\frac{2\mu_N}{R_N} - \frac{\mu_N}{a_{trans}}} - \sqrt{\frac{\mu_N}{R_N}} \longrightarrow V_{\infty,N} = 3931{,}922\ m/s$

$|a| = \frac{\mu_N}{2V_{\infty,N}^2} \longrightarrow \boxed{a = 221103{,}647\ km}$

c) eccentricity and Δt

$e_t = 1 + \frac{r_p V_\infty^2}{\mu_N} = 1 + \frac{(R_N + h)V_{N\infty}^2}{\mu_N} \longrightarrow \boxed{e_{trans} = 1{,}3217}$

0/

FINAL ASTRODYNAMICS

$\boxed{2.}$ geocentric parabolic orbit $\rightarrow e = 1$ $\qquad R_E = 6378$ km

$r_p = 6600$ km $\qquad \mu_\oplus = 3{,}986 \cdot 10^{14}\ m^3/s^2$

a) flight time from $\theta = -90°$ to $\theta = 90°$

$\Delta\theta = 180°$

$p = r_p \cdot 2 = 13200$ km

$$h = \sqrt{p\mu(1+\cos\theta)} = [\theta = 90°] = \sqrt{p\mu} = \sqrt{13200 \cdot 10^3 \cdot 3{,}986 \cdot 10^{14}} \longrightarrow \boxed{h = 7{,}2536 \cdot 10^{10}\ \mathrm{km^2/s}}$$

$$M = \frac{1}{2}\tan\left(\frac{\theta}{2}\right) + \frac{1}{6}\tan^3\left(\frac{\theta}{2}\right) = [\theta = 90°] = \frac{1}{2} + \frac{1}{6} = 2/3$$

$$t = \frac{M h^3}{\mu^2} = \frac{2/3\,(7{,}2536 \cdot 10^{10})^3}{(3{,}986 \cdot 10^{14})^2} = 1{,}6014 \cdot 10^3\ s \rightarrow$$ if t is from $0° \rightarrow 90°$, then for $-90° \rightarrow 90°$ $t_f = 2t$

$$t_f = 2 \cdot t = 2 \cdot 1{,}6014 \cdot 10^3\ s \longrightarrow \boxed{t_f = 53{,}38\ min}$$

1

b) geocentric vector (r, θ) after 24h of perigee

$t = 24h = 86400\ s$

$$M = \frac{\mu^2}{h^3} \cdot t = \frac{(3{,}986 \cdot 10^{14})^2}{(7{,}2536 \cdot 10^{10})^3} \cdot 86400 \longrightarrow \boxed{M = 35{,}9689}$$

$$M = \frac{1}{2}\tan\left(\frac{\theta}{2}\right) + \frac{1}{6}\tan^3\left(\frac{\theta}{2}\right) \rightarrow \tan\left(\frac{\theta}{2}\right) = \left(3M + \sqrt{(3M)^2+1}\right)^{1/3} - \left(3M + \sqrt{(3M)^2+1}\right)^{-1/3}$$

$$\tan\left(\frac{\theta}{2}\right) = \left(3 \cdot 35{,}9689 + \sqrt{(3 \cdot 35{,}9689)^2+1}\right)^{1/3} - \left(3 \cdot 35{,}9689 + \sqrt{(3 \cdot 35{,}9689)^2+1}\right)^{-1/3}$$

$$\tan\left(\frac{\theta}{2}\right) = 5{,}8316 \longrightarrow \theta = 2 \cdot \arctan(5{,}8316) \longrightarrow \boxed{\theta = 160{,}539°}$$

1

$$r = \frac{p}{1+e\cos\theta} = \frac{h^2}{\mu}\,\frac{1}{1+e\cos\theta} = \frac{13200 \cdot 10^3}{1+\cos(160{,}539)} \longrightarrow \boxed{r = 231045{,}871\ \mathrm{Km}}$$

+16 −6

Astrodynamics ESEIAAT UPC
Name: **TR**
DNI or Passport:
Instructions for all attendees:

January 2021
Final Exam

This theoretical part weights 4 points out of 10 in the final exam
There is **ONLY** one correct answer on each question.
Every correctly answered question adds +1.0 point.
Every incorrectly answered question discounts -0.25 points
30 minutes of maximum time to complete the test.
No lecture slides are allowed or Internet resources.

Additional Instructions for remote attendees:
Highlight in **bold** the correct answer.
After completion, **email-me the exam.**
The camera and microphone must be switched on during the entire exam duration.
Only the mouse can be used during the exam to make the text bold.

0) This is a question sample on how to return the exam for remote atendees.
   a) This answer is NOT selected.
   b) This answer is NOT selected.
   c) This answer is selected and this is why it is bolded.
   d) None of the above

1) What is the central assumption in the patched conic method?
   a) To have multiple sphere of influences acting simultaneously
   b) To have only one central body acting on a given time
   c) To have propulsion acting during the cruise phase
   d) None of the above

2) What is a stable time reference, suitable for timekeeping during an entire space mission?
   a) A timescale based on the daily rotation of the Earth
   b) A timescale based on the yearly translation of the Earth around the sun
   c) A timescale based on an atomic oscillator 1
   d) None of the above

3) The inclination of the orbit (assuming a direct insertion without manoeuvring):
   a) Is lower or equal than the latitude of the launch site
   b) Is greater or equal than the latitude of the launch site
   c) Depends on the launch time
   d) None of the above

4) In the two body problem, which of the Keplerian elements are not constant?
   a) True Anomaly, Eccentric Anomaly
   b) Semi-major axis, eccentricity
   c) Inclination, longitude of the ascending node
   d) None of the above

5) Which statement is true before and after a gravity assist manoeuvre:
   a) The velocity of the spacecraft relative to the planet is maintained constant 1
   b) The velocity of the planet relative to the Sun is maintained constant
   c) The velocity of the spacecraft relative to the Sun is maintained constant
   d) None of the above

6) Changing the inclination of the orbital plane is more fuel-efficient:
   a) In the line of nodes, where the spacecraft intersects the plane of reference
   b) At the periapsis, where the velocity is greatest -1
   c) At the apoapsis, where the velocity is lowest
   d) None of the above

7) The quickest orbital transfer, adequate to respond a medical emergency on board is:
   a) Hyperbolic transfer with chemical propulsion
   b) Hohmann transfer with chemical propulsion
   c) Multiple Hohmann segment with multiple chemical low thrust impulses -1
   d) Spiral transfer with continuous electrical propulsion

8) In a restricted circular three body problem, the libration point L3 point is located:
   a) Between the principal mass and the secondary mass, close to the secondary mass
   b) In the line between the primary mass and secondary mass, opposite to the secondary mass 1
   c) +60 degrees from the centre of mass, advanced of the secondary mass
   d) -60 degrees from the centre of mass, retarded of the secondary mass

9) The ascending order of the energy of a Keplerian orbit is:
   a) Circle, Ellipse, Parabola, Hyperbola
   b) Hyperbola, Parabola, Ellipse, Circle 1
   c) Ellipse, Circle, Hyperbola, Parabola
   d) None of the above

10) In a Keplerian orbit representation, the ascending node is:
   a) The line intersecting the orbit plane and plane of reference (e.g. equator) -1
   b) The cross product of the position and velocity vectors
   c) The angle between the orbit plane and a plane of reference (e.g. equator)
   d) None of the above

11) The rotation of Earth presents two clear perturbation periods:
   a) Nutation, with a period of 25,765 years and Precession, with a period of 18.6 years
   b) Polar motion with a period of 28 days and sidereal rotation of 23h56 minutes
   c) Precession, with a period of 25,765 years and Nutation, with a period of 18.6 years 1
   d) Sidereal rotation, with a period of 28 days and Polar motion with a period of 23h56 minutes

12) The effect of the Drag on the orbit
   a) Increase of eccentricity, decrease of perigee height
   b) Decrease of eccentricity, decrease of perigee height
   c) Decrease of eccentricity, decrease of apogee height 1
   d) Increase of eccentricity, decrease of apogee height

13) The period of the Geosynchronous Equatorial Orbit (GEO) is

a) A sidereal day (i.e. 23 h 56 m 4.09 s)
b) A solar day (i.e. 24 h)
c) Proportional to the synodic period
d) None of the above

-1

14) Molniya and Tundra Orbits share

a) The semi-major axis
b) The orbital period
c) The inclination
d) The eccentricity

1

15) The satellite Ground track on a non-rotating spherical Earth

a) It is determined by the intersection of a plane passing through the center of the Earth.
b) Corresponds to a great circle
c) Repeats the same ground track over and over
d) All of the above

1

16) A constellation of satellites is

a) Is composed by satellites of very different design
b) Has a common objective (e.g. communication, navigation, science).
c) Is composed by satellites in different orbit each
d) All of the above

-1

17) The Lambert theorem states that the transfer time of a body moving between two points on a conic trajectory

a) depends on the chord joining these two positions
b) is independent of the direction of motion
c) depends on the conic eccentricity joining these two positions
d) None of the above

1

18) When solving the Lambert problem by Simo's method, the geometric meaning of $\sqrt{z}$ corresponds to

a) The eccentricity of the solution orbit
b) half of the variation in eccentric anomaly between P_1 and P_2 on the solution orbit
c) half of the variation in true anomaly between P_1 and P_2 on the solution orbit
d) None of the above

1

19) Which sentence is not true about the Pork Chop Plot?

a) Depicts the results (e.g. $\Delta V, \Delta\theta$) for various combinations of launch time and time-of-flight Δt
b) Depend on launch constraints such as the range of allowable launch azimuths
c) Provide a preliminary estimate of the amount of propellant to be carried onboard the spacecraft.
d) Every pair of launch time and time-of-flight Δt outputs a different result (e.g. $\Delta V, \Delta\theta$) in the PCP, being no repeated values in the PCP.

1

20) The Broken Plane Manoeuvre is a consequence of

a) Planetary orbits (departure, arrival) are not co-planar
b) Planetary orbits (departure, arrival) are eccentric
c) The low ΔV requirements of polar transfers
d) None of the above

1

21) What is the use of the Tisserand Graph ?

a) To obtain the synodic periods in interplanetary missions
b) To determine whether a mass (e.g. a comet) is the same before/after a gravity assist maneuver
c) To preliminary design interplanetary missions by means of gravity assists
d) None of the above

1

22) In the restricted three-body problem, the third mass (e.g. a spacecraft)

a) Can move freely in any position with any velocity
b) Has forbidden regions that cannot cross into, depending on the value of the Jacobi constant
c) The velocity is zero for any value of the Jacobi constant.
d) None of the above

23) In the n-body problem,

a) The Total Linear Momentum is conserved, as in the two-body problem
b) The Total Angular Momentum is conserved, as in the two-body problem
c) The Total Energy of the system is conserved, as in the two-body problem
d) All of the above

1

24) The Sphere of Influence SOI:

a) Inside the planet SOI, the motion of the Space Vehicle is determined by its equations of motion relative to the Sun
b) Its size is proportional to the mass of the planet versus the Sun
c) Its size is proportional to the distance from the planet to the Sun
d) None of the above

-1

25) Numerical Iterative methods

a) Always convergence to a stable solution with an adequate numerical precision
b) Can require derivatives of first, second or third order
c) The initialization guess must be good enough to assure numerical convergence
d) None of the above

-1

$\frac{3}{5}$ + 2 =

Astrodynamics ESEIAAT UPC

Name: TR

DNI or Passport:

January 2021

Final Exam

Question 1 (4 points):

Compute the ΔV and necessary time to reduce by half the orbital period of a synchronous satellite with Neptune rotation using an inward Hohman transfer.

Assuming

$$\mu_N = 6.836529 \cdot 10^{15}\, m^3/s^2$$

$$R_N = 24764\, km$$

a) Assuming an orbit inclination of cero degrees and a circular orbit, determine the orbit height at which the orbiter would be synchronised with the Neptune's sidereal period of $16.11\, h$
b) Determine the final semi-major axis of the final semi-synchronous orbit ($i.e.\ \mathrm{T_f} = \mathrm{T_0}/2$)
c) For the Hohman transfer, compute the eccentricity and the time required to perform the manoeuvre:
d) Determine the initial, final and total required ΔV, plotting (sketching) where each ΔV takes places, and its direction:
e) Assuming a solid chemical propulsion system of $\mathrm{I_{sp}} = 300$ s, compute the required mass fraction of propellant to perform such maneuver.

Question 2 (2 points):

A geocentric parabolic orbit has a perigee radius of $6600\, km$

Assuming

$$\mathrm{R_E} = 6378\ \mathrm{km}$$
$$\mu_\mathrm{E} = 3.986 \cdot 10^{14}\ m^3/s^2$$

a) Determine the flight time from $\theta = -90^\circ$ to $\theta = +90^\circ$
b) Determine the geocentric vector (r, θ) after 24 h of orbiting the perigee.

QUESTION 2

$r_p = 6.600 km.$

$R_E = 6378 km$

$\mu_E = 3.986 \cdot 10^{14} m^3/s^2$

a) $r_p = 6{,}6 \cdot 10^6 m \longrightarrow p = r_p \cdot 2 = 13'2 \cdot 10^6 m$

$h = \sqrt{p \cdot \mu} = 7'2536 \cdot 10^{10}$

$t \to \theta_1 = -90°, \theta_2 = 90° \to \Delta\theta = 180°$

$t_1 \Rightarrow 0-90° \Rightarrow r = \frac{P}{1 \cos\theta} = 25'956 \cdot 10^6 m$

$M = \frac{1}{2} tg\left(\frac{\theta}{2}\right) + \frac{1}{6} tg^3\left(\frac{\theta}{2}\right) = \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$

$t_1 = \frac{M \cdot h^3}{\mu^2} = 1'6014 \cdot 10^3 s$

$\boxed{t_t = 2t_1 = 3_0 202'8\cancel{3} s = 6'8877 h.}$

b) $t = 24h = 86400s$

$M = \frac{\mu^2}{h^3} \cdot t = 35.8607$

$\theta = 2 \cdot atg\left((3M + \sqrt{(3M)^2+1})^{(1/3)} - (3M + \sqrt{(3M)^2+1})^{(-1/3)} \right)$

$\theta = 2{,}8016 \text{ rad} \longrightarrow 160{,}52°$

$r = \frac{P}{1+\cos\theta} = 2{,}3057 \cdot 10^8 m$

$(r, \theta) = (2{,}3057 \cdot 10^8 m, 160{,}52°)$

QUESTION 1

a) $i = 0$

circular orbit $\rightarrow e = 0$

$h? \longrightarrow$ syncro with $T = 16.11h. \rightarrow$

$$T = \frac{2\pi}{n}$$

$$n = \sqrt{\mu / a^3} \qquad T = \frac{2\pi}{\sqrt{\frac{\mu}{a^3}}}$$

OK!

$$\sqrt{\frac{\mu}{a^3}} = \frac{2\pi}{T}$$

$(R_N + h) =$

$$\frac{\mu}{a^3} = \left(\frac{2\pi}{T}\right)^2$$

$R_N + h = a$

$h = a - R_N$

$$a^3 = \frac{\mu}{\left(\frac{2\pi}{T}\right)^2}$$

$h = \boxed{5,875 \cdot 10^7 m}$

1

$$a = \sqrt[3]{\frac{\mu}{\left(\frac{2\pi}{T}\right)^2}} = \cancel{7,637 \cdot 10^{11} m}$$

$8,3514 \cdot 10^7 m.$

b) $T_f = T_0/2 = 28998 s.$

$$a_H = \sqrt[3]{\frac{\mu_N}{\left(\frac{2\pi}{T_f}\right)^2}} = 5,261 \cdot 10^7 m$$

1

c) $r_p = a(1-e) =$

$$-e = \cancel{\frac{r_p}{a} + 1} \qquad -e = \frac{r_p}{a} - 1$$

$$\frac{r_p}{a} = (1-e) \qquad e = -\left(\frac{r_p}{a} - 1\right)$$

$$e = -\left(\frac{R_N + h}{a_H} - 1\right)$$

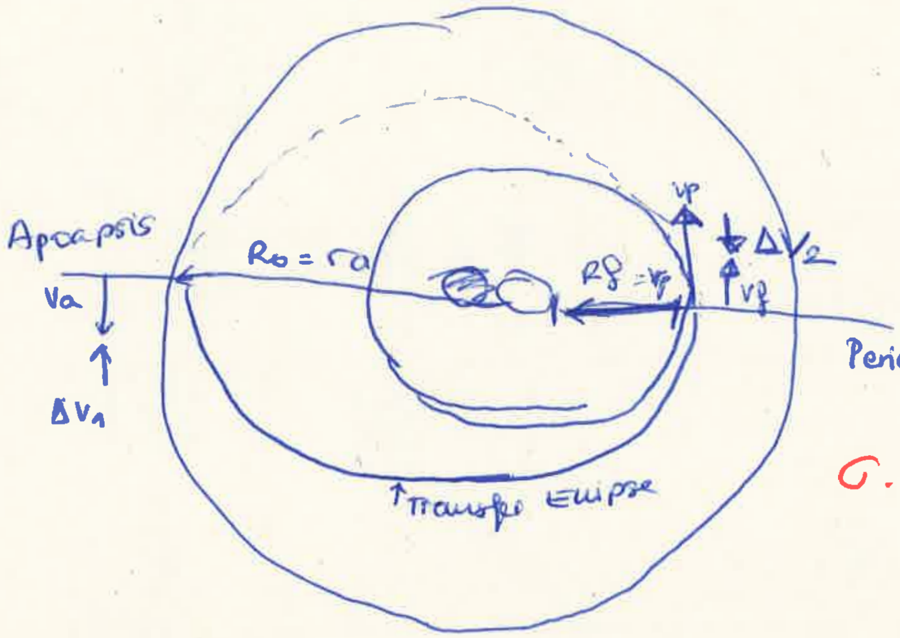
c) e and Δt $\quad \Delta V = |\Delta V_1| + |\Delta V_2|$ $\quad$ ∅

e) $$\frac{\Delta m}{m_0} = 1 - e^{\frac{\Delta V}{I_{sp} \cdot g_{sL}}}$$

És aplicar la fórmula si sabes ΔV i a.

0.5.

d)



2 tangencial impulses at $\theta = 0$ and $\theta = 180°$

0.5.

+11 -10

Astrodynamics ESEIAAT UPC
Name: UG
DNI or Passport:
Instructions for all attendees:

January 2021
Final Exam

This theoretical part weights 4 points out of 10 in the final exam
There is **ONLY** one correct answer on each question.
Every correctly answered question adds +1.0 point.
Every incorrectly answered question discounts -0.25 points
30 minutes of maximum time to complete the test.
No lecture slides are allowed or Internet resources.

Additional Instructions for remote attendees:
Highlight in **bold** the correct answer.
After completion, **email-me the exam.**
The camera and microphone must be switched on during the entire exam duration.
Only the mouse can be used during the exam to make the text bold.

0) This is a question sample on how to return the exam for remote atendees.
   a) This answer is NOT selected.
   b) This answer is NOT selected.
   c) This answer is selected and this is why it is bolded.
   d) None of the above

1) What is the central assumption in the patched conic method?
   a) To have multiple sphere of influences acting simultaneously
   b) To have only one central body acting on a given time — 1
   c) To have propulsion acting during the cruise phase
   d) None of the above

2) What is a stable time reference, suitable for timekeeping during an entire space mission?
   a) A timescale based on the daily rotation of the Earth
   b) A timescale based on the yearly translation of the Earth around the sun
   c) A timescale based on an atomic oscillator 1
   d) None of the above

3) The inclination of the orbit (assuming a direct insertion without manoeuvring):
   a) Is lower or equal than the latitude of the launch site
   b) Is greater or equal than the latitude of the launch site 1
   c) Depends on the launch time
   d) None of the above

4) In the two body problem, which of the Keplerian elements are not constant?
   a) True Anomaly, Eccentric Anomaly
   b) Semi-major axis, eccentricity
   c) Inclination, longitude of the ascending node
   d) None of the above

5) Which statement is true before and after a gravity assist manoeuvre:

   a) The velocity of the spacecraft relative to the planet is maintained constant
   b) The velocity of the planet relative to the Sun is maintained constant -1
   c) The velocity of the spacecraft relative to the Sun is maintained constant
   d) None of the above

6) Changing the inclination of the orbital plane is more fuel-efficient:

   a) In the line of nodes, where the spacecraft intersects the plane of reference -1
   b) At the periapsis, where the velocity is greatest
   c) At the apoapsis, where the velocity is lowest
   d) None of the above

7) The quickest orbital transfer, adequate to respond a medical emergency on board is:

   a) Hyperbolic transfer with chemical propulsion
   b) Hohmann transfer with chemical propulsion 1
   c) Multiple Hohmann segment with multiple chemical low thrust impulses
   d) Spiral transfer with continuous electrical propulsion

8) In a restricted circular three body problem, the libration point L3 point is located:

   a) Between the principal mass and the secondary mass, close to the secondary mass
   b) In the line between the primary mass and secondary mass, opposite to the secondary mass 1
   c) +60 degrees from the centre of mass, advanced of the secondary mass
   d) -60 degrees from the centre of mass, retarded of the secondary mass

9) The ascending order of the energy of a Keplerian orbit is:

   a) Circle, Ellipse, Parabola, Hyperbola
   b) Hyperbola, Parabola, Ellipse, Circle -1
   c) Ellipse, Circle, Hyperbola, Parabola
   d) None of the above

10) In a Keplerian orbit representation, the ascending node is:

   a) The line intersecting the orbit plane and plane of reference (e.g. equator)
   b) The cross product of the position and velocity vectors -1
   c) The angle between the orbit plane and a plane of reference (e.g. equator)
   d) None of the above

11) The rotation of Earth presents two clear perturbation periods:

   a) Nutation, with a period of 25,765 years and Precession, with a period of 18.6 years
   b) Polar motion with a period of 28 days and sidereal rotation of 23h56 minutes
   c) Precession, with a period of 25,765 years and Nutation, with a period of 18.6 years 1
   d) Sidereal rotation, with a period of 28 days and Polar motion with a period of 23h56 minutes

12) The effect of the Drag on the orbit

   a) Increase of eccentricity, decrease of perigee height
   b) Decrease of eccentricity, decrease of perigee height
   c) Decrease of eccentricity, decrease of apogee height -1
   d) Increase of eccentricity, decrease of apogee height

$e = \frac{\sqrt{a^2 - b^2}}{a}$

13) The period of the Geosynchronous Equatorial Orbit (GEO) is
   a) A sidereal day (i.e. 23 h 56 m 4.09 s)
   b) A solar day (i.e. 24 h)
   c) Proportional to the synodic period 1
   d) None of the above

14) Molniya and Tundra Orbits share
   a) The semi-major axis
   b) The orbital period
   c) The inclination 1
   d) The eccentricity

15) The satellite Ground track on a non-rotating spherical Earth
   a) It is determined by the intersection of a plane passing through the center of the Earth.
   b) Corresponds to a great circle
   c) Repeats the same ground track over and over
   d) All of the above

16) A constellation of satellites is
   a) Is composed by satellites of very different design
   b) Has a common objective (e.g. communication, navigation, science). -1
   c) Is composed by satellites in different orbit each
   d) All of the above

17) The Lambert theorem states that the transfer time of a body moving between two points on a conic trajectory
   a) depends on the chord joining these two positions
   b) is independent of the direction of motion
   c) depends on the conic eccentricity joining these two positions -1
   d) None of the above

18) When solving the Lambert problem by Simo's method, the geometric meaning of $\sqrt{z}$ corresponds to
   a) The eccentricity of the solution orbit
   b) half of the variation in eccentric anomaly between P_1 and P_2 on the solution orbit
   c) half of the variation in true anomaly between P_1 and P_2 on the solution orbit -1
   d) None of the above

19) Which sentence is not true about the Pork Chop Plot?
   a) Depicts the results (e.g. $\Delta V, \Delta\theta$) for various combinations of launch time and time-of-flight Δt
   b) Depend on launch constraints such as the range of allowable launch azimuths
   c) Provide a preliminary estimate of the amount of propellant to be carried onboard the spacecraft.
   d) Every pair of launch time and time-of-flight Δt outputs a different result (e.g. $\Delta V, \Delta\theta$) in the PCP, being no repeated values in the PCP. 1

20) The Broken Plane Manoeuvre is a consequence of

a) Planetary orbits (departure, arrival) are not co-planar
b) Planetary orbits (departure, arrival) are eccentric
c) The low ΔV requirements of polar transfers
d) None of the above

21) What is the use of the Tisserand Graph ?

a) To obtain the synodic periods in interplanetary missions
b) To determine whether a mass (e.g. a comet) is the same before/after a gravity assist maneuver
c) To preliminary design interplanetary missions by means of gravity assists
d) None of the above

-1

22) In the restricted three-body problem, the third mass (e.g. a spacecraft)

a) Can move freely in any position with any velocity
b) Has forbidden regions that cannot cross into, depending on the value of the Jacobi constant
c) The velocity is zero for any value of the Jacobi constant.
d) None of the above

23) In the n-body problem,

a) The Total Linear Momentum is conserved, as in the two-body problem
b) The Total Angular Momentum is conserved, as in the two-body problem
c) The Total Energy of the system is conserved, as in the two-body problem
d) All of the above

1

24) The Sphere of Influence SOI:

a) Inside the planet SOI, the motion of the Space Vehicle is determined by its equations of motion relative to the Sun
b) Its size is proportional to the mass of the planet versus the Sun
c) Its size is proportional to the distance from the planet to the Sun
d) None of the above

1

25) Numerical Iterative methods

a) Always convergence to a stable solution with an adequate numerical precision
b) Can require derivatives of first, second or third order
c) The initialization guess must be good enough to assure numerical convergence
d) None of the above

1

4+2 = 6

Astrodynamics ESEIAAT UPC
Name: UG
DNI or Passport:
Instructions for all attendees:

January 2021
Final Exam

Compute the ΔV and necessary time to reduce by half the orbital period of a synchronous satellite with Neptune rotation using an inward Hohman transfer.

Assuming

$$\mu_N = 6.836529 \cdot 10^{15}\ m^3/s^2$$

$$R_N = 24764\ km$$

a) Assuming an orbit inclination of cero degrees and a circular orbit, determine the orbit height at which the orbiter would be synchronised with the Neptune's sidereal period of 16.11 h
b) Determine the final semi-major axis of the final semi-synchronous orbit ($i.e.\ \mathrm{T_f} = \mathrm{T_0}/2$)
c) For the Hohman transfer, compute the eccentricity and the time required to perform the manoeuvre:
d) Determine the initial, final and total required ΔV, plotting (sketching) where each ΔV takes places, and its direction:
e) Assuming a solid chemical propulsion system of $\mathrm{I_{sp}} = 300$ s, compute the required mass fraction of propellant to perform such maneuver.

Question 2 (2 points):

A geocentric parabolic orbit has a perigee radius of 6600 km

Assuming

$$\mathrm{R_E} = 6378\ \mathrm{km}$$
$$\mu_\mathrm{E} = 3.986 \cdot 10^{14}\ m^3/s^2$$

a) Determine the flight time from $\theta = -90^\circ$ to $\theta = +90^\circ$
b) Determine the geocentric vector (r, θ) after 24 h of orbiting the perigee.

Question 2

$r_p = 6'6 \cdot 10^6$ m $\quad R_E = 6'378 \cdot 10^6$ m $\quad \mu_E = 3'986 \cdot 10^{14}$ m^3/s^2

a) $\Delta t_{-90^\circ \to 90^\circ}$? Because of the symmetry of the problem,

$\Delta t_{-90^\circ \to 90^\circ} = 2 \cdot \Delta t_{0^\circ - 90^\circ}$

$$\cancel{\theta = 90^\circ \rightarrow r = \frac{p}{1+\cos(90)} =}$$

$$\cancel{p = r_p 2 = 13'2 \cdot 10^6}$$

$p = r_p \cdot 2 = 13'2 \cdot 10^6$ m $\quad h = \sqrt{p \cdot \mu} = 7'2536 \cdot 10$ m^2/s

$$M_{\theta=90^\circ} = \frac{1}{2} \cdot \cancelto{1}{\tan\left(\frac{90}{2}\right)} + \frac{1}{3} \cancelto{1}{\tan^3\left(\frac{90}{2}\right)} = \frac{2}{3}$$

$M_{\theta=0^\circ} = 0 \qquad \Delta M = \frac{2}{3} \qquad \Delta t_1 = \dfrac{\Delta M \cdot h^3}{\mu^2} = 1601'382$ s

$$\boxed{t_t = 2 \cdot \Delta t_1 = 3202'764 \text{ s} = 0'8897 \text{ h}}$$

b) $t = 24\text{h} \longrightarrow r?\ \theta?$

$t = 86400$ s

$M = \dfrac{\mu^2}{h^3} t = 35'9689$

$$\theta = 2 \cdot \operatorname{atan}\left[\left(3M + \sqrt{(3M)^2+1}\right)^{1/3} - \left(3M + \sqrt{(3M)^2+1}\right)^{-1/3}\right]$$

$$\boxed{\theta = 160'5392^\circ}$$

$$\boxed{r = \frac{p}{1+\cos\theta} = 231050'574 \text{ km}}$$

Question 1

$\mu_N = 6'836529 \cdot 10^{15} \; m^3/s^2$ $\quad R_N = 24764 km$

a) $i=0$, $e=0$ $\quad T = 16'11\,h \Rightarrow h?$

$$T = 2\pi\sqrt{\frac{a^3}{\mu}} = 16'11h = 57996 s$$

$$57996 = 2\pi\sqrt{\frac{a^3}{6'836529\cdot10^{15}}} \Longrightarrow a = 83513'625 \, km = r_0$$

$$\boxed{h = r - R_N = 58749'625 \, km}$$

b) $T_f = \frac{T_0}{2} = 8'055h = 28998 s$

$$28998s = 2\pi\sqrt{\frac{a^3}{\mu}} \Rightarrow \boxed{a = 52610'287 \, km} = r_f$$

c) $$\boxed{e_{trans} = \frac{r_0 - r_f}{r_0 + r_f} = 0'227}$$

$a_{trans} = 68061'956 \, km$

$$\boxed{\Delta t_{trans} = \frac{T_{trans}}{2} = \pi\sqrt{\frac{a_{trans}^3}{\mu_N}} = 21334'8s = 5'926h}$$

d) ΔV_1? ΔV_2? ΔV?

$$V_0 = \sqrt{\frac{\mu}{r_0}} = 9047'72 \, m/s$$

$$V_a = \sqrt{\frac{2\mu}{r_0} - \frac{\mu}{a_{trans}}} = \cancel{5811'7}\; 7954'67 \, m/s$$

$$V_p = \sqrt{\frac{2\mu}{r_f} - \frac{\mu}{a_{trans}}} = \cancel{11399'41}\; 12627'25 \, m/s$$

$$V_f = \sqrt{\frac{\mu}{r_f}} = 11399'41 \, m/s$$

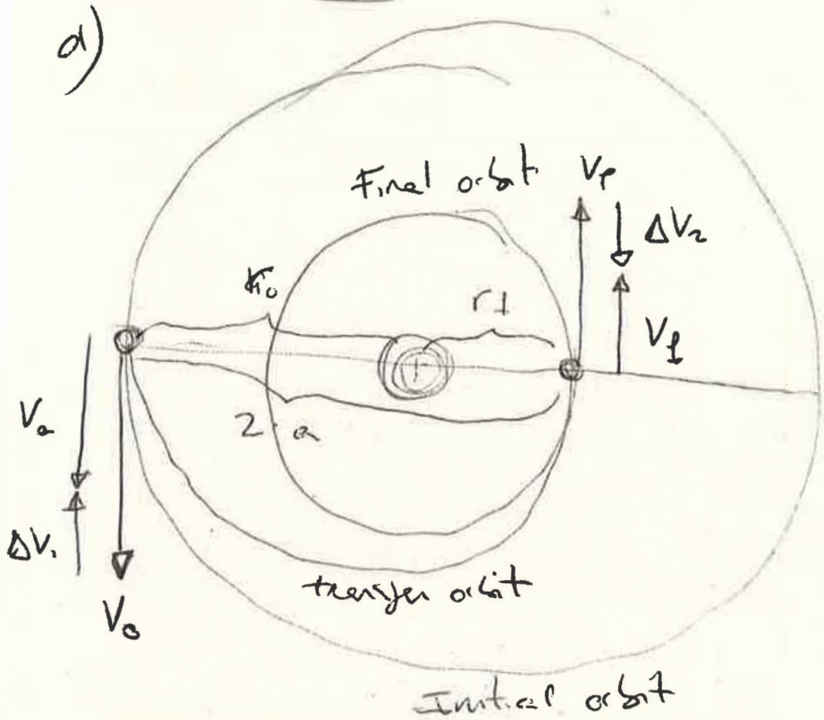
$$\boxed{\Delta V_1 = V_a - V_0 = -1093'05 \, m/s}$$

$$\boxed{\Delta V_2 = -1227'84 \, m/s}$$

$$\boxed{\Delta V_3 = |\Delta V_1| + |\Delta V_2| = 2320'89}$$

Question 1

d)



e) $I_{sp} = 300\ s$, solid

$$\boxed{\frac{\Delta m}{m_0} = 1 - e^{-\frac{\Delta V}{I_{sp} \cdot g_{SL}}} = 0'5455}$$

1

+24 -1

Astrodynamics ESEIAAT UPC
Name: PM
DNI or Passport:
Instructions for all attendees:

January 2021
Final Exam

This theoretical part weights 4 points out of 10 in the final exam
There is **ONLY** one correct answer on each question.
Every correctly answered question adds +1.0 point.
Every incorrectly answered question discounts -0.25 points
30 minutes of maximum time to complete the test.
No lecture slides are allowed or Internet resources.

Additional Instructions for remote attendees:
Highlight in **bold** the correct answer.
After completion, **email-me the exam.**
The camera and microphone must be switched on during the entire exam duration.
Only the mouse can be used during the exam to make the text bold.

0) This is a question sample on how to return the exam for remote atendees.
   a) This answer is NOT selected.
   b) This answer is NOT selected.
   c) This answer is selected and this is why it is bolded.
   d) None of the above

1) What is the central assumption in the patched conic method?
   a) To have multiple sphere of influences acting simultaneously
   b) To have only one central body acting on a given time 1
   c) To have propulsion acting during the cruise phase
   d) None of the above

2) What is a stable time reference, suitable for timekeeping during an entire space mission?
   a) A timescale based on the daily rotation of the Earth
   b) A timescale based on the yearly translation of the Earth around the sun 1
   c) A timescale based on an atomic oscillator
   d) None of the above

3) The inclination of the orbit (assuming a direct insertion without manoeuvring):
   a) Is lower or equal than the latitude of the launch site
   b) Is greater or equal than the latitude of the launch site -1
   c) Depends on the launch time
   d) None of the above

4) In the two body problem, which of the Keplerian elements are not constant?
   a) True Anomaly, Eccentric Anomaly
   b) Semi-major axis, eccentricity 1
   c) Inclination, longitude of the ascending node
   d) None of the above

5) Which statement is true before and after a gravity assist manoeuvre:

a) The velocity of the spacecraft relative to the planet is maintained constant 1

NOK b) The velocity of the planet relative to the Sun is maintained constant

c) The velocity of the spacecraft relative to the Sun is maintained constant

d) None of the above

6) Changing the inclination of the orbital plane is more fuel-efficient:

a) In the line of nodes, where the spacecraft intersects the plane of reference

b) At the periapsis, where the velocity is greatest -1

c) At the apoapsis, where the velocity is lowest

d) None of the above

7) The quickest orbital transfer, adequate to respond a medical emergency on board is:

a) Hyperbolic transfer with chemical propulsion

b) Hohmann transfer with chemical propulsion -1

c) Multiple Hohmann segment with multiple chemical low thrust impulses

d) Spiral transfer with continuous electrical propulsion

8) In a restricted circular three body problem, the libration point L3 point is located:

a) Between the principal mass and the secondary mass, close to the secondary mass 1

b) In the line between the primary mass and secondary mass, opposite to the secondary mass

c) +60 degrees from the centre of mass, advanced of the secondary mass

d) -60 degrees from the centre of mass, retarded of the secondary mass

9) The ascending order of the energy of a Keplerian orbit is:

a) Circle, Ellipse, Parabola, Hyperbola

b) Hyperbola, Parabola, Ellipse, Circle 1

c) Ellipse, Circle, Hyperbola, Parabola

d) None of the above

10) In a Keplerian orbit representation, the ascending node is:

a) The line intersecting the orbit plane and plane of reference (e.g. equator)

b) The cross product of the position and velocity vectors 1

c) The angle between the orbit plane and a plane of reference (e.g. equator)

d) None of the above

11) The rotation of Earth presents two clear perturbation periods:

a) Nutation, with a period of 25,765 years and Precession, with a period of 18.6 years

b) Polar motion with a period of 28 days and sidereal rotation of 23h56 minutes

c) Precession, with a period of 25,765 years and Nutation, with a period of 18.6 years 1

d) Sidereal rotation, with a period of 28 days and Polar motion with a period of 23h56 minutes

12) The effect of the Drag on the orbit

a) Increase of eccentricity, decrease of perigee height

b) Decrease of eccentricity, decrease of perigee height

c) Decrease of eccentricity, decrease of apogee height 1

d) Increase of eccentricity, decrease of apogee height

13) The period of the Geosynchronous Equatorial Orbit (GEO) is

a) A sidereal day (i.e. 23 h 56 m 4.09 s)
b) A solar day (i.e. 24 h)
c) Proportional to the synodic period
d) None of the above

1

14) Molniya and Tundra Orbits share

a) The semi-major axis
b) The orbital period
c) The inclination
d) The eccentricity

1

15) The satellite Ground track on a non-rotating spherical Earth

a) It is determined by the intersection of a plane passing through the center of the Earth.
b) Corresponds to a great circle
c) Repeats the same ground track over and over
d) All of the above

1

16) A constellation of satellites is

a) Is composed by satellites of very different design
b) Has a common objective (e.g. communication, navigation, science).
c) Is composed by satellites in different orbit each
d) All of the above

1

17) The Lambert theorem states that the transfer time of a body moving between two points on a conic trajectory

a) depends on the chord joining these two positions
b) is independent of the direction of motion
c) depends on the conic eccentricity joining these two positions
d) None of the above

1

18) When solving the Lambert problem by Simo's method, the geometric meaning of $\sqrt{z}$ corresponds to

a) The eccentricity of the solution orbit
b) half of the variation in eccentric anomaly between P_1 and P_2 on the solution orbit
c) half of the variation in true anomaly between P_1 and P_2 on the solution orbit
d) None of the above

1

19) Which sentence is not true about the Pork Chop Plot?

a) Depicts the results (e.g. $\Delta V, \Delta\theta$) for various combinations of launch time and time-of-flight Δt
b) Depend on launch constraints such as the range of allowable launch azimuths
c) Provide a preliminary estimate of the amount of propellant to be carried onboard the spacecraft.
d) Every pair of launch time and time-of-flight Δt outputs a different result (e.g. $\Delta V, \Delta\theta$) in the PCP, being no repeated values in the PCP.

1

20) The Broken Plane Manoeuvre is a consequence of

a) Planetary orbits (departure, arrival) are not co-planar
b) Planetary orbits (departure, arrival) are eccentric
c) The low ΔV requirements of polar transfers
d) None of the above

1

21) What is the use of the Tisserand Graph ?

a) To obtain the synodic periods in interplanetary missions
b) To determine whether a mass (e.g. a comet) is the same before/after a gravity assist maneuver
c) To preliminary design interplanetary missions by means of gravity assists
d) None of the above

1

22) In the restricted three-body problem, the third mass (e.g. a spacecraft)

a) Can move freely in any position with any velocity
b) Has forbidden regions that cannot cross into, depending on the value of the Jacobi constant
c) The velocity is zero for any value of the Jacobi constant.
d) None of the above

1

23) In the n-body problem,

a) The Total Linear Momentum is conserved, as in the two-body problem
b) The Total Angular Momentum is conserved, as in the two-body problem
c) The Total Energy of the system is conserved, as in the two-body problem
d) All of the above

1

24) The Sphere of Influence SOI:

a) Inside the planet SOI, the motion of the Space Vehicle is determined by its equations of motion relative to the Sun
b) Its size is proportional to the mass of the planet versus the Sun
c) Its size is proportional to the distance from the planet to the Sun
d) None of the above

-1

25) Numerical Iterative methods

a) Always convergence to a stable solution with an adequate numerical precision
b) Can require derivatives of first, second or third order
c) The initialization guess must be good enough to assure numerical convergence
d) None of the above

1

4 + 2 = 6

Astrodynamics ESEIAAT UPC

January 2021

Name: PM

Final Exam

DNI or Passport:

Question 1 (4 points):

Compute the ΔV and necessary time to reduce by half the orbital period of a synchronous satellite with Neptune rotation using an inward Hohman transfer.

Assuming

$$\mu_N = 6.836529 \cdot 10^{15}\ m^3/s^2$$

$$R_N = 24764\ km$$

a) Assuming an orbit inclination of cero degrees and a circular orbit, determine the orbit height at which the orbiter would be synchronised with the Neptune's sidereal period of 16.11 h
b) Determine the final semi-major axis of the final semi-synchronous orbit ($i.e.\ \mathrm{T_f} = \mathrm{T_0}/2$)
c) For the Hohman transfer, compute the eccentricity and the time required to perform the manoeuvre:
d) Determine the initial, final and total required ΔV, plotting (sketching) where each ΔV takes places, and its direction:
e) Assuming a solid chemical propulsion system of $\mathrm{I_{sp}} = 300$ s, compute the required mass fraction of propellant to perform such maneuver.

Question 2 (2 points):

A geocentric parabolic orbit has a perigee radius of 6600 km

Assuming

$$\mathrm{R_E} = 6378\ \mathrm{km}$$
$$\mu_\mathrm{E} = 3.986 \cdot 10^{14}\ m^3/s^2$$

a) Determine the flight time from $\theta = -90°$ to $\theta = +90°$
b) Determine the geocentric vector (r, θ) after 24 h of orbiting the perigee.

Question 1

a)

$$T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

To be synchronized $\rightarrow$ $T = 16'11h = 57996\,s$

Hence,

$$57996 = 2\pi\sqrt{\frac{a^3}{\mu_N}} \rightarrow \sqrt[3]{\left(\frac{57996}{2\pi}\right)^2 \cdot \mu_N} = a \rightarrow$$

$$\rightarrow a = \sqrt[3]{\left(\frac{57996}{2\pi}\right)^2 \cdot 6'836529 \cdot 10^{15}} = 83513625'04\,m$$

As it's a circular orbit $\rightarrow$ $r = cte = a$

Then,

$$h = r - R_N = 83513625'04 - 24764000 = 58749625'04\,m$$

$$\boxed{h(T = 16'11h) = 58749625'04\,m}$$

b)

$T_0 = 57996\,s$

$T_f = \frac{T_0}{2} = 28998\,s$

Thus,

$$a_f = \sqrt[3]{\left(\frac{T_f}{2\pi}\right)^2 \mu_N} = \sqrt[3]{\left(\frac{28998}{2\pi}\right)^2 \cdot 6'836529 \cdot 10^{15}} = 52610287'07\,m$$

$$\boxed{a_f = 52610287'07\,m}$$

c)

From the slides it is known that the eccentricity of an original Hohmann transfer orbit is:

$$e_{trans} = \frac{r_0 - r_f}{r_0 + r_f} \xrightarrow[\text{are circles}]{\text{the initial \& final orbits}} e_{trans} = \frac{a_0 - a_f}{a_0 + a_f}$$

Thus,

$$e_{trans} = \frac{83513625'04 - 52610287'07}{83513625'04 + 52610287'07}$$

$$\boxed{e_{trans} = 0'227072} \rightarrow \text{Elliptical orbit, as expected}$$

The time required to perform the manoeuvre will be half the period of the Hohmann transfer orbit; hence:

$$\Delta t_{trans} = \frac{T_{trans}}{2} = \pi\sqrt{\frac{a_{trans}^3}{\mu_N}}$$

$$\downarrow a_{trans} = \frac{a_0 + a_f}{2} = 68061956'06 m$$

Thus,

$$\Delta t_{trans} = \pi\sqrt{\frac{68061956'06^3}{6'836529 \cdot 10^{15}}} = 21334'8 s =$$

$$= 5'926 h$$

$$\boxed{\Delta t_{trans} = 5'926 h}$$

2

Question 1

d)

$$\Delta V_1 = V_a - V_o = \sqrt{\frac{2\cdot\mu_N}{a_o} - \frac{\mu_N}{a_{tran}}} - \sqrt{\frac{\mu_N}{a_o}}$$

$$\Delta V_1 = \sqrt{\frac{2\cdot 6'836529\cdot 10^{15}}{83513625'04} - \frac{6'836529\cdot 10^{15}}{68061956'06}} - \sqrt{\frac{6'836529\cdot 10^{15}}{83513625'04}}$$

$$\Delta V_1 = 7954'67\ \frac{m}{s} - 9047'72\ \frac{m}{s} = -1093'05\ \frac{m}{s}$$

$$\boxed{|\Delta V_1| = 1093'05\ \frac{m}{s}}$$ → Initial ΔV

$$\Delta V_2 = V_f - V_p = \sqrt{\frac{\mu}{a_f}} - \sqrt{\frac{2\mu_N}{a_f} - \frac{\mu_N}{a_{tran}}}$$

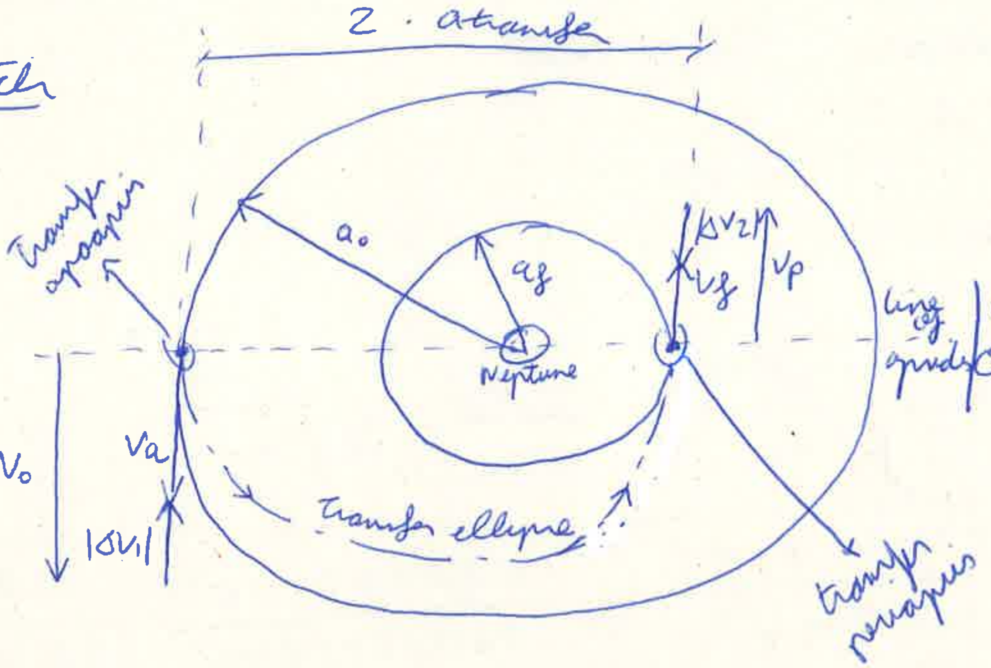
$$\Delta V_2 = \sqrt{\frac{6'836529\cdot 10^{15}}{52610287'07}} - \sqrt{\frac{2\cdot 6'836529\cdot 10^{15}}{52610287'07} - \frac{6'836529\cdot 10^{15}}{68061956'06}}$$

$$\Delta V_2 = 11399'41\ \frac{m}{s} - 12627'25\ \frac{m}{s} = -1227'84\ \frac{m}{s}$$

$$\boxed{|\Delta V_2| = 1227'84\ \frac{m}{s}}$$ → Final ΔV

$$\boxed{|\Delta V_T| = |\Delta V_1| + |\Delta V_2| = 2320'89\ \frac{m}{s}}$$ → Total ΔV

Sketch



$\Delta\theta = 180°$

line of apsides $\theta_0 = 180°$, $\theta_f = 0°$ with respect to transfer ellipse

e)

From the course's slides:

$$\frac{\Delta m}{m_0} = 1 - e^{-\frac{\Delta V}{I_{sp} \cdot g_{SL}}}$$

Propellant mass fraction

Then, as $I_{sp} = 300\,s$, $\Delta V = |\Delta V_T| = 2320'89 \frac{m}{s}$, and g_{SL} is $9'81 \frac{m}{s^2}$:

$$\frac{\Delta m}{m_0} = 1 - e^{-\frac{2320'89}{300 \cdot 9'81}} = 0'5455$$

~~Per tant, la~~

Then, the propellant mass fraction for this manoeuvre is: $\boxed{\frac{\Delta m}{m_0} = 0'5453}$ ~~(més de la meitat del pes inicial tirada~~ (more than half of the initial mass will have to be propellant)

9

Question 2

$r_p = 6600000 m$

$R_E = 6378000 m$

$\mu_E = 3'986 \cdot 10^{14} \frac{m^3}{s^2}$

a) $\theta_0 = -90°$ & $\theta_f = 90°$

$M_0 = \frac{1}{2} tg\left(\frac{\theta_0}{2}\right) + \frac{1}{6} tg\left(\frac{\theta_0}{2}\right)^3$

$M_0 = \frac{1}{2} tg(-45) + \frac{1}{6} tg(-45)^3 = -\frac{2}{3}$

$M_f = \frac{1}{2} tg\left(\frac{\theta_f}{2}\right) + \frac{1}{6} tg\left(\frac{\theta_f}{2}\right)^3$

$M_f = \frac{1}{2} tg(45) + \frac{1}{6} tg(45)^3 = \frac{2}{3}$

$r_p = \frac{h^2}{\mu_E} \cdot \frac{1}{1+\cos(0)} \rightarrow \sqrt{2 r_p \mu_E} = h =$

$= \sqrt{2 \cdot 6600000 \cdot 3'986 \cdot 10^{14}} = 7'254 \cdot 10^{10} \frac{m^2}{s}$

Then,

$\frac{\mu^2}{h^3} \Delta t = \Delta M \rightarrow \Delta t = \frac{h^3}{\mu^2} \Delta M = \frac{(7'254 \cdot 10^{10})^3}{(3'986 \cdot 10^{14})^2}\left(\frac{2}{3} + \frac{2}{3}\right)$

$\Delta t = 3203'29 s = 0'88 98 h$

$\boxed{\Delta t = 0'8898 h}$

⑤

b) $\theta_0 = 0$; $\theta_f = ?$

$$\Delta M = M_f - \cancel{M_0}^{0 \text{ (perigee)}} = \frac{\mu^2}{h^3} (t_f - \cancel{t_0}^{0 \text{ (perigee)}})$$

$$M_f = \frac{(3'986 \cdot 10^{14})^2}{(7'254 \cdot 10^{10})^3} (24 \cdot 3600) = 35'96$$

From the course's slides,

$$\theta_f = 2 \operatorname{atan} \left[\left(3 M_f + \sqrt{(3m)^2 + 1}\right)^{1/3} - \left(3 M_f + \sqrt{(3 M_f)^2 + 1}\right)^{-1/3} \right]$$

$$\theta_f = 2 \operatorname{atan} (5'831) \hat{=} 160'54^\circ$$

Then,

$$r_f = \frac{h^2}{\mu_E} \frac{1}{1 + \cos \theta_f} = \frac{(7'254 \cdot 10^{10})^2}{3'986 \cdot 10^{14}} \cdot \frac{1}{1 + \cos (160'54)}$$

$$r_f = r(\theta = 160'54^\circ) = 231092734'9 \text{ m}$$

Thus, the geocentric vector after 24h orbiting is

$$\boxed{(r, \theta) = (231092734'9 m, 160'54^\circ)}$$